

Blow-up rates for semi-linear reaction–diffusion systems

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Abstract

Let $\Omega \subset \mathbb{R}^n$ ($n \geq 1$) be a bounded smooth domain. Consider the following initial–boundary value problem of reaction–diffusion systems

$$\begin{aligned}
 \partial_t u_1 - \Delta u_1 - u_1^{p_{11}} u_2^{p_{12}} &= 0, & \text{in } (0, T) \times \Omega, \\
 \partial_t u_2 - \Delta u_2 - u_1^{p_{21}} u_2^{p_{22}} &= 0, & \text{in } (0, T) \times \Omega, \\
 \mathbf{u}(t, x) &= 0, & \text{on } (0, T) \times \partial\Omega, \\
 \mathbf{u}(0, x) = \Phi(x) &\geq 0, & \text{in } \Omega,
 \end{aligned} \tag{I}$$

where $\mathbf{u} = (u_1, u_2) \geq 0$, and $\Phi(x) = (\varphi_1(x), \varphi_2(x)) \geq 0$, and ν is the unit outer normal at $\partial\Omega$ and $T \in (0, \infty]$ is the maximum existence time of $\mathbf{u}$ (in L^∞ -norm) and the exponents p_{ij} , $i, j = 1, 2$, are non-negative real numbers.

Systems of form (I) naturally arise in studying non-linear phenomena in biology, chemistry, medicine and physics. For instance, (I) has been used to model densities and temperatures in chemical reactions, condensate amplitudes in Bose–Einstein condensates, wave amplitudes (or envelopes of multiple interacting optical modes) in optical fibers, and pattern formation in ecological systems.

Under suitable conditions on p_{ij} , $i, j = 1, 2$, we established the following *exact* blow-up rates for blow-up solutions $\mathbf{u}$ of (I)

$$C(T - t)^{-\theta_i} \leq \sup_{x \in \Omega} u_i(t, x) \leq C^{-1}(T - t)^{-\theta_i}, \quad i = 1, 2,$$

where θ_1 and $\theta_2 > 0$ are positive exponents depending only on p_{ij} , generalizing earlier results in this direction.

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1. Introduction

Let $\Omega \subset \mathbb{R}^n$ ($n \geq 1$) be a bounded smooth domain. Consider the following initial–boundary value problem of reaction–diffusion systems

$$\begin{aligned} \partial_t u_1 - \Delta u_1 - u_1^{p_{11}} u_2^{p_{12}} &= 0, & \text{in } (0, T) \times \Omega, \\ \partial_t u_2 - \Delta u_2 - u_1^{p_{21}} u_2^{p_{22}} &= 0, & \text{in } (0, T) \times \Omega, \\ \mathbf{u}(t, x) &= 0, & \text{on } (0, T) \times \partial\Omega, \\ \mathbf{u}(0, x) &= \Phi(x) \geq 0, & \text{in } \Omega, \end{aligned} \quad (1.1)$$

where $\mathbf{u} = (u_1, u_2) \geq 0$, and $\Phi(x) = (\varphi_1(x), \varphi_2(x)) \geq 0$,¹ and v is the unit outer normal at $\partial\Omega$ and $T \in (0, \infty]$ is the maximum existence time of $\mathbf{u}$ (in L^∞ -norm) and the exponents p_{ij} , $i, j = 1, 2$, are non-negative real numbers (it is understood $0^0 = 1$). Throughout the paper and without further mentioning, we assume

$$p_{12}p_{21} > 0.$$

That is, system (1.1) is *irreducible*.

Systems of form (1.1) naturally arise in studying non-linear phenomena in biology, chemistry and physics. For instance, (1.1) has been used to model densities and temperatures in chemical reactions, condensate amplitudes in Bose–Einstein condensates, wave amplitudes (or envelopes of multiple interacting optical modes) in optical fibers, and pattern formation in ecological systems. The quantities u_i 's represent densities, temperatures, amplitudes, etc., and are non-negative, see [12,13,15,16] and references therein.

A non-negative solution $\mathbf{u}$ of (1.1) is called a *global* solution if its maximal existence time T in L^∞ -norm is infinite, i.e., $T = \infty$. While $\mathbf{u}$ is called a *blow-up* solution if $T \in (0, \infty)$ and there holds

$$\lim_{t \rightarrow T^-} \max_{i=1,2} \sup_{x \in \Omega} u_i(t, x) = \infty.$$

Then T is called the blow-up time of $\mathbf{u}$. It is well-known, as in the scalar case, (1.1) admits *both* global and blow-up solutions (under suitable conditions), see for example [1,3,5,6,9,17] and the references therein. In this paper we are interested in establishing *exact* blow-up rates for blow-up solutions $\mathbf{u}$ of (1.1). That is, our goal is to derive the following estimates

$$C(T-t)^{-\theta_i} \leq \sup_{x \in \Omega} u_i(t, x) \leq C^{-1}(T-t)^{-\theta_i}, \quad i = 1, 2 \text{ and } t \in (0, T), \quad (1.2)$$

¹ Bold face letters are used to denote vectors and all relations involving vectors are understood in the component-wise sense.

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