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Approximate controllability of a class of semilinear degenerate systems with boundary control

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Abstract

This paper concerns a class of control systems governed by semilinear degenerate equations with boundary control in one-dimensional space. The control is proposed on the 'degenerate' part of the boundary. The control systems are shown to be approximately controllable by Kakutani's fixed point theorem.

MSC: 93B05; 93C20; 35K65

Keywords: Approximate controllability; Degenerate equation; Boundary control

1. Introduction

In this paper, we investigate the approximate controllability of the system governed by the following semilinear degenerate equations:

$$u_t - \left(x^\alpha u_x\right)_x + g(x,t,u) = 0, \quad (x,t) \in Q_T, \tag{1.1}$$

$$u(0,t) = h(t)\chi_{[T_1,T_2]}, \qquad u(1,t) = 0, \quad t \in (0,T), \tag{1.2}$$

$$u(x,0) = u_0(x), \quad x \in (0,1), \tag{1.3}$$

where $0 < \alpha < 1$, $Q_T = (0,T) \times (0,1)$, h is the control function, χ is the characteristic function, $0 < T_1 < T_2 < T$, while g is a measurable function satisfying

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$$\left|g(x,t,u)-g(x,t,v)\right| \leqslant M|u-v|, \quad (x,t)\in Q_T,\ u,v\in\mathbb{R}$$

for some $M>0$. Moreover, it is assumed that $g(\cdot,\cdot,0)\in L^2(Q_T)$ and $g(x,t,u)$ is differentiable at $u=0$ uniformly in Q_T. The approximate controllability problem is formulated as follows: for any $u_d\in L^2(\Omega)$ and any admissible error value $\varepsilon>0$, whether there exists a control function h such that the solution u to the problem (1.1)–(1.3) approximately approaches u_d at time T, i.e.

$$\left\|u(\cdot,T)-u_d(\cdot)\right\|_{L^2(\Omega)} \leqslant \varepsilon. \tag{1.4}$$

The degenerate equation (1.1) can be used to describe some physical models. For instance, we can find a motivating example of a Crocco-type equation coming from the study of the velocity field of a laminar flow on a flat plate (see [19]).

The controllability of degenerate parabolic systems has been well studied. For example, for the problem

$$\begin{gathered} w_t-\left(x^\alpha w_x\right)_x+g(x,t,u)=h(x,t)\chi_\omega, \quad (x,t)\in Q_T, \\ w(0,t)=0 \quad \text{if } 0<\alpha<1, \qquad \left(x^\alpha w_x\right)(0,t)=0 \quad \text{if } \alpha\geqslant 1, \quad t\in(0,T), \\ w(1,t)=0, \quad t\in(0,T), \\ w(x,0)=w_0(x), \quad x\in(0,1), \end{gathered}$$

where $\omega=(a,b)$ is a nonempty subset of $(0,1)$ and $h\in L^2(Q_T)$ is the control function, it is shown that it is null controllable if $0<\alpha<2$ [1,3,7,8,20], while not if $\alpha\geqslant 2$ [9]. Although the system may be not null controllable, Cannarsa et al. [9] proved the regional and the persistent regional null controllability, and Wang [21] showed the approximate controllability for each $\alpha>0$. Generalizations of the above result to nondivergence form and the equation with semilinear lower order terms are also studied in [4,5] and [6,10,14,22], respectively.

From the controllability for the nondegenerate parabolic operators [2,13,15], we know that if a system is controllable when the control acts in the interior of the domain, then it is also controllable when the control acts on the boundary. However, for degenerate operators, there is no null controllability result when the control acts on 'degenerate' part of the boundary. As for the approximate controllability, there are a few papers that considered this case. In [11], the authors showed the approximate controllability of the problem

$$u_t-\left(x^\alpha u_x\right)_x=0, \quad (x,t)\in Q_T, \tag{1.5}$$

$$u(0,t)=h(t)\chi_{[T_1,T_2]}, \qquad u(1,t)=0, \quad t\in(0,T), \tag{1.6}$$

$$u(x,0)=u_0(x), \quad x\in(0,1), \tag{1.7}$$

where $0<\alpha<1$ and $h\in H_0^1(0,T)$. As is well known, the approximate controllability of (1.5)–(1.7) is equivalent to the unique continuation for the adjoint equation

$$v_t+\left(x^\alpha v_x\right)_x=0. \tag{1.8}$$