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Convergence of the Ostrovsky equation to the Ostrovsky–Hunter one ☆

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Abstract

We consider the Ostrovsky equation, which contains nonlinear dispersive effects. We prove that as the diffusion parameter tends to zero, the solutions of the dispersive equation converge to discontinuous weak solutions of the Ostrovsky–Hunter equation. The proof relies on deriving suitable a priori estimates together with an application of the compensated compactness method in the L^p setting.

MSC: 35G25; 35L65; 35L05

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1. Introduction

The nonlinear evolution equation

$$\partial_x\left(\partial_t u + u\partial_x u - \beta\partial^3_{xxx} u\right) = \gamma u, \tag{1.1}$$

with β and $\gamma \in \mathbb{R}$ was derived by Ostrovsky [18] to model small-amplitude long waves in a rotating fluid of a finite depth. This equation generalizes the Korteweg–deVries equation (that

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corresponds to $\gamma = 0$) by the additional term induced by the Coriolis force. Mathematical properties of the Ostrovsky equation (1.1) were studied recently in many detail, including the local and global well-posedness in energy space [6,11,14,23], stability of solitary waves [9,12,15], and convergence of solutions in the limit of the Korteweg–deVries equation [10,15]. We rewrite (1.1) in the following way

$$\begin{cases} \partial_t u + u\partial_x u - \beta\partial_{xxx}^3 u = \gamma \displaystyle\int_0^x u(t,y)\,dy, & t > 0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1.2}$$

or equivalently,

$$\begin{cases} \partial_t u + u\partial_x u - \beta\partial_{xxx}^3 u = \gamma P, & t > 0,\ x \in \mathbb{R}, \\ \partial_x P = u, & t > 0,\ x \in \mathbb{R}, \\ P(t,0) = 0, & t > 0, \\ u(0,x) = u_0(x), & x \in \mathbb{R}. \end{cases} \tag{1.3}$$

We are interested in the no high frequency limit, i.e., we send $\beta \to 0$ in (1.1). In this way we pass from (1.1) to the equation

$$\begin{cases} \partial_x(\partial_t u + u\partial_x u) = \gamma u, & t > 0,\ x \in \mathbb{R}, \\ u(0,x) = u_0(x), & x \in \mathbb{R}. \end{cases} \tag{1.4}$$

Eq. (1.4) is known under different names such as the reduced Ostrovsky equation [19,22], the Ostrovsky–Hunter equation [1], the short-wave equation [7], and the Vakhnenko equation [16, 20]. Integrating (1.4) with respect to x we gain the integro-differential formulation of (1.4) (see [13])

$$\partial_t u + \partial_x\left(\frac{u^2}{2}\right) = \gamma \int_0^x u(t,y)\,dy,$$

that is equivalent to

$$\partial_t u + \partial_x\left(\frac{u^2}{2}\right) = \gamma P, \qquad \partial_x P = u, \qquad P(\cdot,0) = 0, \qquad u(0,\cdot) = u_0. \tag{1.5}$$

On the initial datum, we assume that

$$u_0 \in L^2(\mathbb{R}) \cap L^4(\mathbb{R}), \qquad \int_{\mathbb{R}} u_0(x)\,dx = 0, \tag{1.6}$$

and on the function

$$P_0(x) = \int_{-\infty}^x u_0(y)\,dy, \quad x \in \mathbb{R}, \tag{1.7}$$