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On a quasilinear parabolic–elliptic chemotaxis system with logistic source

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Abstract

This paper deals with a quasilinear parabolic–elliptic chemotaxis system with logistic source, under homogeneous Neumann boundary conditions in a smooth bounded domain. For the case of positive diffusion function, it is shown that the corresponding initial boundary value problem possesses a unique global classical solution which is uniformly bounded. Moreover, if the diffusion function is zero at some point, or a positive diffusion function and the logistic damping effect is rather mild, we proved that the weak solutions are global existence. Finally, it is asserted that the solutions approach constant equilibria in the large time for a specific case of the logistic source.

MSC: 92C17; 35K55; 35B40

Keywords: Chemotaxis; Global existence; Boundedness; Logistic source

1. Introduction

In this paper, we consider the following quasilinear parabolic–elliptic chemotaxis system with logistic source

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$$\begin{cases} u_t = \nabla \cdot \big(D(u)\nabla u\big) - \chi \nabla \cdot (u\nabla v) + f(u), & x \in \Omega,\ t > 0, \\ 0 = \Delta v - v + u, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$ and $\partial/\partial\nu$ denotes the derivative with respect to the outer normal of $\partial\Omega$, $\chi > 0$ is a parameter referred to as chemosensitivity, and $u = u(x,t)$ denotes the density of the cells population and $v = v(x,t)$ represents the concentration of the chemoattractant. The function $f : [0,\infty) \mapsto \mathbb{R}$ is smooth and satisfies $f(0) \geqslant 0$ as well as

$$f(u) \leqslant a - bu^{\gamma} \quad \text{for all } u \geqslant 0 \tag{1.2}$$

with some $a \geqslant 0$, $b > 0$ and $\gamma > 1$. Moreover, in some places in addition we will require a corresponding lower estimate

$$f(u) \geqslant -a_0\big(u^{\gamma} + 1\big) \quad \text{for all } u \geqslant 0 \tag{1.3}$$

with some $a_0 > 0$.

Chemotaxis is the directed movement of the cells as a response to gradients of the concentration of the chemical signal substance in their environment. The biased movement is referred to as chemoattraction if the cells move toward the increasing signal concentration, while it is called chemorepulsion whenever the cells move away from the increasing signal concentration. The origin of chemotaxis model was introduced by Keller and Segel [18] and we refer the reader to the paper [13] where a comprehensive information of further examples illustrating the outstanding biological relevance of chemotaxis can be found.

During the past four decades, the chemotaxis models have become one of the best study models in mathematical biology, and throughout the main issue of the investigation was whether the chemotaxis model allows for a chemotactic collapse, that is, if it possesses solutions that blow up in finite or infinite time. However, in this present paper, we will derive the uniformly boundedness of model (1.1) under some assumptions on the diffusion function and the logistic source, which rules out the chemotactic collapse.

In order to better understand model (1.1), let us mention some previous contributions in this direction. For example, the following initial boundary value problem has been studied by many authors

$$\begin{cases} u_t = \nabla \cdot \big(D(u)\nabla u\big) - \nabla \cdot \big(S(u)\nabla v\big) + f(u), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega,\ t > 0. \end{cases} \tag{1.4}$$

In the absence of the logistic source (i.e. $f(u) \equiv 0$) for model (1.4), the knowledge appears to be rather complete. For instance, in the case of $D(u) \equiv 1$, the problem has been studied extensively by many authors [39,12,15,16,40,14,42]. Horstmann and Winkler [15] showed that the solutions are global and bounded provided that $S(u) \leqslant c(u+1)^{\frac{2}{n}-\epsilon}$ for all $u \geqslant 0$ with some $\epsilon > 0$ and $c > 0$, while if $S(u) \geqslant c(u+1)^{\frac{2}{n}+\epsilon}$ for all $u \geqslant 0$ with $\epsilon > 0$, $c > 0$ and $n \geqslant 2$, and some further technical conditions are satisfied, then the radial blow-up solutions were constructed. Moreover, for more general $D(u)$, many results concerning the question whether the solutions are bounded or blow up [3,6–8,14,5,19,28,37]. For example, if $\frac{S(u)}{D(u)} \geqslant c(u+1)^{\frac{2}{n}+\epsilon}$ for all $u \geqslant 0$