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The Armstrong–Frederick cyclic hardening plasticity model with Cosserat effects

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Abstract

We propose an extension of the cyclic hardening plasticity model formulated by Armstrong and Frederick which includes micropolar effects. Our micropolar extension establishes coercivity of the model which is otherwise not present. We study then existence of solutions to the quasistatic, rate-independent Armstrong–Frederick model with Cosserat effects which is, however, still of non-monotone, non-associated type. In order to do this, we need to relax the pointwise definition of the flow rule into a suitable weak energy-type inequality. It is shown that the limit in the Yosida approximation process satisfies this new solution concept. The limit functions have a better regularity than previously known in the literature, where the original Armstrong–Frederick model has been studied.

1. Introduction

One of the well-known models to describe cyclic plasticity is the non-linear kinematic hardening model formulated by Armstrong and Frederick [3]. This model has been highly rated, because it is based on a physical mechanism of strain hardening and dynamic recovery, and because it

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has the capability of representing reasonably well the shapes of stress–strain hysteresis loops, especially those of constant strain ranges. Therefore, implementation of the Armstrong–Frederick model in finite element methods has been examined in several studies to date. Thus, that model is now available as a material model of cyclic hardening plasticity in commercial, general-purpose software for finite element analysis.

The Armstrong–Frederick model (AF) is a modification of the Melan–Prager model, which is well known in the literature and it can also be seen as an approximation of the Prandtl–Reuss model. The key modification of this simple model is the so-called "recall"-term, changing the evolution law for the symmetric backstress tensor b from a classical linear kinematic hardening law (Melan–Prager) to a nonlinear kinematic hardening law, i.e.,

$$b_t = \underbrace{c\,\varepsilon_t^p}_{\text{lin. kin. hardening}} - \underbrace{d\,|\varepsilon_t^p|b}_{\text{recall-term, nonlinear hardening}}.$$

Here, ε^p is the symmetric plastic strain tensor, c and d are positive material constants. The "recall"-term entails the L^∞-boundedness of the backstress b, a property which is an experimental fact since to the contrary, for high frequency cycles softening and rupture will occur. Therefore, the AF-model shows nonlinear kinematic hardening, but only to within a certain extent. The more realistic description of cyclic hardening plasticity experiments with the AF-model, however, has a prize to be paid: the model is non-coercive (bounded hardening), it is of non-monotone type and not of gradient type (non-associated flow rule). Thus, the AF-model is one of the prominent small strain plasticity models which has yet escaped the efforts of mathematicians to establish well-posedness.

The mathematical analysis being quite challenging, there are no encompassing existence results for this model in the literature. The first (partial) mathematical result for the Armstrong–Frederick model was obtained by the first author in the article [10]. There, the non-monotone, non-associated AF-model was written as a model of pre-monotone type (for the definition we refer to [2]). In this work the author used a Yosida approximation to the monotone part of the flow rule. The obtained a priori estimates are, however, not sufficient to pass to the limit with such approximations and to get L^2-strong solutions (see Section 3 in [10]). It was only shown that the limit functions satisfy the so-called "reduced energy inequality". In the article [19] a regularization of the "recall"-term in the equation for the backstress was proposed. The existence of a rescaled in time solution to the Armstrong–Frederick model with the regularized equation for the backstress could then be established. The rescaling idea is very simple: a new time variable $\tau = \zeta(t)$ is proposed. Then the new system is easier to analyze, because the plastic strain is now uniformly Lipschitz with respect to the rescaled time. The main problem is to get back to the original system with the rescaled in time solution. It is, in principle, possible for rate-independent models, where the flow rule is invariant under scaling of the time variable. The Armstrong–Frederick model is rate-independent but the authors of [19] are not able to get back to the original system. The rescaling idea has already been applied in the plasticity context in [4,16,17].

In this paper we want to extend the system of equations proposed by Armstrong and Frederick to include micropolar effects. In the classical metal perfect plasticity models at infinitesimal strains it has been shown in a series of papers [11–13,24] that a coupling with Cosserat elasticity may also regularize the ill-posedness of the Prandtl–Reuss plasticity model. This is possible because the Cosserat coupling leads to coercivity. Perfect plasticity, however, is yet characterized by a monotone flow rule of gradient type (associated plasticity). Therefore,