



# On the blow-up criterion and small data global existence for the Hall-magnetohydrodynamics

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## Abstract

In this paper, we establish an optimal blow-up criterion for classical solutions to the incompressible resistive Hall-magnetohydrodynamic equations. We also prove two global-in-time existence results of the classical solutions for small initial data, the smallness conditions of which are given by the suitable Sobolev and the Besov norms respectively. Although the Sobolev space version is already an improvement of the corresponding result in [4], the optimality in terms of the scaling property is achieved via the Besov space estimate. The special property of the energy estimate in terms of  $\dot{B}_{2,1}^s$  norm is essential for this result. Contrary to the usual MHD the global well-posedness in the  $2\frac{1}{2}$  dimensional Hall-MHD is wide open.  
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## 1. Introduction

We study the following three dimensional incompressible resistive viscous Hall-magnetohydrodynamics system (Hall-MHD):

$$\partial_t u + (u \cdot \nabla)u + \nabla p = (\nabla \times B) \times B + \Delta u, \quad (1.1)$$

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$$\nabla \cdot u = 0, \quad \text{in } \mathbb{R}^3 \times (0, \infty), \quad (1.2)$$

$$\partial_t B - \nabla \times (u \times B) + \nabla \times ((\nabla \times B) \times B) = \Delta B, \quad (1.3)$$

$$(u(0, x), B(0, x)) = (u_0(x), B_0(x)), \quad \text{in } \mathbb{R}^3, \quad (1.4)$$

where  $u$ ,  $B$  and  $p$  represent 3-dimensional velocity vector field, the magnetic field and scalar pressure, respectively. The initial data  $u_0$  and  $B_0$  satisfy

$$\nabla \cdot u_0 = \nabla \cdot B_0 = 0.$$

Note that if  $\nabla \cdot B_0 = 0$ , then the divergence free condition is propagated by (1.3). Comparing with the usual MHD equations, the Hall-MHD equations have the Hall term  $\nabla \times ((\nabla \times B) \times B)$  in (1.3), which plays an important role in magnetic reconnection which is happening in the case of large magnetic shear. The Hall magnetohydrodynamics was studied systematically by Lighthill [11]. In particular, he considered Alfvén waves with Hall effect. The Hall-MHD is important describing many physical phenomena, e.g., space plasmas, star formation, neutron stars and geo-dynamo (see [2,7,8,13,16,18] and references therein).

In [1], Acheritogaray, Degond, Frouvelle and Liu derived the Hall-MHD equations from either two fluids model or kinetic models in a mathematically rigorous way. In [4], the global existence of weak solutions to (1.1)–(1.4) as well as the local well-posedness of classical solution are established. Also, a blow-up criterion for smooth solution to (1.1)–(1.4) and the global existence of smooth solution for small initial data are obtained (see [4, Theorems 2.2 and 2.3]). Very recently, temporal decay for the weak solution and smooth solution with small data to Hall-MHD are also established in [6].

Our goal of this paper is to improve in an optimal way the blow-up criterion and global existence of smooth solution with small initial data to the Hall-MHD equations (1.1)–(1.4) derived in [4]. The sense of optimality is explained in detail in Remark 2.

Using vector identity, we can rewrite (1.1)–(1.4) as follows:

$$\partial_t u + (u \cdot \nabla)u + \nabla \left( p + \frac{|B|^2}{2} \right) = (B \cdot \nabla)B + \Delta u, \quad (1.5)$$

$$\nabla \cdot u = 0, \quad \text{in } \mathbb{R}^3 \times (0, \infty), \quad (1.6)$$

$$\partial_t B + (u \cdot \nabla)B + \nabla \times ((\nabla \times B) \times B) = (B \cdot \nabla)u + \Delta B, \quad (1.7)$$

$$(u(0, x), B(0, x)) = (u_0(x), B_0(x)), \quad \text{in } \mathbb{R}^3. \quad (1.8)$$

Note that a weak solution  $(u, B)$  to (1.1)–(1.4) satisfies the following energy inequality (see [4]):

$$\begin{aligned} & \frac{1}{2} (\|u(t)\|_{L^2}^2 + \|B(t)\|_{L^2}^2) + \int_0^t \|\nabla u(\cdot, \tau)\|_{L^2}^2 + \|\nabla B(\cdot, \tau)\|_{L^2}^2 d\tau \\ & \leq \frac{1}{2} (\|u_0\|_{L^2}^2 + \|B_0\|_{L^2}^2) \end{aligned} \quad (1.9)$$

for almost every  $t \in [0, \infty)$ .

Our first result is Serrin type [15] criterion for the solutions to (1.1)–(1.4).

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