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Asymptotic behavior in degenerate parabolic fully nonlinear equations and its application to elliptic eigenvalue problems

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ABSTRACT

We study the fully nonlinear parabolic equation

$$F\left(D^2 u^m\right) - u_t = 0 \quad \text{in } \Omega \times (0, +\infty),\ m \geqslant 1,$$

with the Dirichlet boundary condition and positive initial data in a smooth bounded domain $\Omega \subset \mathbb{R}^n$, provided that the operator F is uniformly elliptic and positively homogeneous of order one. We prove that the renormalized limit of parabolic flow $u(x,t)$ as $t \to +\infty$ is the corresponding positive eigenfunction which solves

$$F\left(D^2 \varphi\right) + \mu \varphi^p = 0 \quad \text{in } \Omega,$$

where $0 < p := \frac{1}{m} \leqslant 1$ and $\mu > 0$ is the corresponding eigenvalue. We also show that some geometric property of the positive initial data is preserved by the parabolic flow, under the additional assumptions that Ω is convex and F is concave. As a consequence, the positive eigenfunction has such geometric property, that is, $\log(\varphi)$ is concave in the case $p = 1$, and $\varphi^{\frac{1-p}{2}}$ is concave for $0 < p < 1$.

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1. Introduction

In this paper, we consider the asymptotic behavior of the solution u to the following fully nonlinear uniformly or degenerate parabolic equation

$$\begin{cases} F(D^2 u^m) - \partial_t u = 0 & \text{in } \Omega \times (0, +\infty), \\ u(\cdot, 0) > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \times (0, +\infty), \end{cases} \tag{1.1}$$

in the range of the exponents $m \geqslant 1$, where we assume that F is uniformly elliptic and positively homogeneous of order one and Ω is a smooth bounded domain in $\mathbb{R}^n$. When F is the Laplace operator, (1.1) is the well-known heat equation for $m = 1$, porous medium equation for $m > 1$, and fast diffusion equation for $0 < m < 1$, respectively. For the Laplace operator, the asymptotic behavior of solutions to the degenerate or singular diffusion equations has been studied by many authors (Aronson, Berryman, Bonforte, Carrillo, Friedman, Galaktionov, Holland, Kamin, Kwong, Peletier, Toscani, Vazquez et al.). We refer to [22–24] and references therein.

In this work, we first show that a renormalized flow of $u(x, t)$ with nonnegative initial data converges to $\varphi(x)$ uniformly for $x \in \Omega$ as $t \to +\infty$, which solves the following elliptic eigenvalue problem

$$\begin{cases} F(D^2 \varphi) + \mu \varphi^p = 0 & \text{in } \Omega, \\ \varphi > 0 & \text{in } \Omega, \\ \varphi = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

where $0 < p := \frac{1}{m} \leqslant 1$ and $\mu > 0$ is the corresponding eigenvalue depending on m. The existence and uniqueness up to a multiplicative constant of the solution to the fully nonlinear eigenvalue problem (1.2) for $p = 1$ were proven by Ishii and Yoshimura [14], and they also showed that the principal eigenvalue $\mu > 0$ is unique. The simplified proof can be found at [1]. Recently, the principal eigenvalues of more general fully nonlinear operators in bounded domains have been studied by many authors and we refer to [1] and references therein. For the sub-linear case $0 < p < 1$, the unique positive eigenfunction of (1.2) can be established by using a barrier argument as in Theorem 3.4.

The asymptotic convergence of renormalized solutions of (1.1) to the problem (1.2) was considered in [23] for the Laplace operator in the range of $0 < p < p_s$, where p_s is the Sobolev critical exponent ($p_s := \frac{n+2}{n-2}$ for $m \geqslant 3$, infinity for $n = 1, 2$). It has been also extended to some fully nonlinear operator F in [17] with the super-linear exponents $1 < p < \mathrm{p}_{\Omega,F}$ (for some critical number $\mathrm{p}_{\Omega,F} > 1$). Since a positive eigenfunction of super-linear case can be approximated by a renormalized solution of the fast diffusion equation, which will extinct in finite time, the Harnack type estimate of the solutions

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