J. Differential Equations 253 (2012) 1287–1316

Contents lists available at SciVerse ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

Insensitizing controls for a class of quasilinear parabolic equations

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ARTICLE INFO

Article history:
Received 13 May 2011
Revised 9 May 2012
Available online 28 May 2012

Keywords:
Insensitizing control
Null controllability
Quasilinear parabolic equation

ABSTRACT

This paper is addressed to showing the existence of insensitizing controls for a class of quasilinear parabolic equations with homogeneous Dirichlet boundary conditions. As usual, this insensitizing problem is reduced to a nonstandard null controllability problem of some nonlinear cascade system governed by a quasilinear parabolic equation and a linear parabolic equation. Nevertheless, in order to solve the later quasilinear controllability problem by the fixed point technique, we need to establish the null controllability of the linearized cascade parabolic system in the framework of classical solutions. The key point is to find the desired control function in a Hölder space for given data with certain regularities.

1. Introduction and main result

Let $n \in \mathbb{N}\setminus\{0\}$, $T > 0$, and let Ω be a bounded domain in $\mathbb{R}^n$ with a C^3 boundary Γ. Put $Q = \Omega \times (0,T)$ and $\Sigma = \Gamma \times (0,T)$. Assume ω and $\mathcal{O}$ to be two given nonempty open subsets of Ω. Denote by χ_ω the characteristic function of the set ω. We consider the following controlled quasilinear parabolic equation:

$$\begin{cases} y_t - \sum_{i,j=1}^{n} \left(a^{ij}(y) y_{x_i}\right)_{x_j} + f(y) = \xi + \chi_\omega u & \text{in } Q, \\ y = 0 & \text{on } \Sigma, \\ y(0) = y_0 + \tau \hat{y}_0 & \text{in } \Omega, \end{cases} \tag{1.1}$$

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[1] This work is supported by the NSF of China under grants 10901032, 11171060 and 11071036, and by the National Basic Research Program of China (973 Program) under grant 2011CB808002.

0022-0396/$ – see front matter
http://dx.doi.org/10.1016/j.jde.2012.05.018

where $x \equiv (x_1, \dots, x_n)$ and t are respectively the space variable and the time variable, y and u are respectively the state variable and the control variable, ξ and y_0 are two known functions, τ is an unknown small real number, $\hat{y}_0$ is an unknown function, f is a given C^2 function defined on $\mathbb{R}$ with $f(0) = 0$, and $a^{ij}(\cdot) : \mathbb{R} \to \mathbb{R}$ are given C^3 functions satisfying $a^{ij} = a^{ji}$ $(i, j = 1, \dots, n)$ and for some constant $\rho > 0$,

$$\sum_{i,j=1}^{n} a^{ij}(s)\eta_i\eta_j \geqslant \rho|\eta|^2, \quad \forall (s, \eta) = (s, \eta_1, \dots, \eta_n) \in \mathbb{R} \times \mathbb{R}^n. \tag{1.2}$$

Let us define the following (partial) energy functional:

$$\Phi(y) = \frac{1}{2} \int_0^T \int_{\mathcal{O}} \left| y(x, t; \tau, u) \right|^2 dx\,dt, \tag{1.3}$$

where $y = y(x, t; \tau, u)$ is the corresponding solution of (1.1) associated to τ and u. In this paper, we are interested in the existence of a control function u (depending on ξ and y_0 but independent of τ and $\hat{y}_0$), which makes the above functional Φ be insensitive with respect to small perturbations on the initial value y_0. A physical interpretation of this problem is as follows: If the state variable y stands for the temperature of a body, then Eq. (1.1) describes the heat conduction of the body, while the diffusion coefficients depend on the temperature in a manner as $a^{ij}(y)$. In (1.1), ξ can be viewed as a given heat source acting on the body, and one can also act on a local domain ω of the body by means of a heat source u. Roughly speaking, the insensitivity problem means that we are expected to find a local heat source u such that the local energy Φ in $\mathcal{O}$ is almost invariant with respect to small perturbations on the initial temperature.

To be more precise, we need to introduce some notations. For any $k, \ell \in \mathbb{N}$, we denote by $C^{k,\ell}(\overline{Q})$ the set of all functions having continuous derivatives in $\overline{Q}$ up to order k with respect to the space variable and up to order ℓ with respect to the time variable, and by $C^k(\overline{\Omega})$ the set of all functions having continuous derivatives in $\overline{\Omega}$ up to order k. For any $\theta \in (0, 1)$, write

$$C^{k+\theta, \ell+\frac{\theta}{2}}(\overline{Q}) = \left\{ u \in C^{k,\ell}(\overline{Q});\ \sup_{|\sigma|=k} \sup_{(x_1,t_1)\neq(x_2,t_2)} \frac{|\partial_x^\sigma \partial_t^\ell u(x_1, t_1) - \partial_x^\sigma \partial_t^\ell u(x_2, t_2)|}{(|x_1 - x_2| + |t_1 - t_2|^{1/2})^\theta} < \infty \right\},$$

and

$$C^{2+\theta}(\overline{\Omega}) = \left\{ u \in C^2(\overline{\Omega});\ \sup_{|\sigma|=2} \sup_{x_1 \neq x_2} \frac{|\partial_x^\sigma u(x_1) - \partial_x^\sigma u(x_2)|}{|x_1 - x_2|^\theta} < \infty \right\},$$

both of which are Banach spaces endowed with the canonical norms.

Since we are treating a nonlinear problem, for given functions $\xi \in C^{\theta, \frac{\theta}{2}}(\overline{Q})$ and $y_0 \in C^{2+\theta}(\overline{\Omega})$ satisfying suitable conditions (which will be specified later), we require that the desired insensitizing control $u(\in C^{\theta, \frac{\theta}{2}}(\overline{Q}))$, which depends on ξ and y_0 but is independent of τ and $\hat{y}_0$, satisfies the following condition:

(H) *There exists a $\tau_0 > 0$ such that for any $|\tau| < \tau_0$ and any $\hat{y}_0 \in C_0^\infty(\Omega)$ with $|\hat{y}_0|_{C^{2+\theta}(\overline{\Omega})} = 1$, Eq.* (1.1) *admits a unique solution $y(\cdot, \cdot; \tau, u) \in C^{2+\theta, 1+\frac{\theta}{2}}(\overline{Q})$. Moreover,*

$$|y|_{C^{2+\theta, 1+\frac{\theta}{2}}(\overline{Q})} \leqslant C\big(n, \Omega, \Gamma, T, a^{ij}, f\big)\big(|\xi|_{C^{\theta, \frac{\theta}{2}}(\overline{Q})} + |u|_{C^{\theta, \frac{\theta}{2}}(\overline{Q})} + |y_0 + \tau \hat{y}_0|_{C^{2+\theta}(\overline{\Omega})}\big). \tag{1.4}$$