



Sturm–Liouville boundary value problems with operator potentials and unitary equivalence

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ABSTRACT

Consider the minimal Sturm–Liouville operator $A = A_{\min}$ generated by the differential expression

$$\mathcal{A} := -\frac{d^2}{dt^2} + T$$

in the Hilbert space $L^2(\mathbb{R}_+, \mathcal{H})$ where $T = T^* \geq 0$ in \mathcal{H} . We investigate the absolutely continuous parts of different self-adjoint realizations of \mathcal{A} . In particular, we show that Dirichlet and Neumann realizations, A^D and A^N , are absolutely continuous and unitary equivalent to each other and to the absolutely continuous part of the Krein realization. Moreover, if $\inf \sigma_{\text{ess}}(T) = \inf \sigma(T)$, then the part $\tilde{A}^{ac} E_{\tilde{A}}(\sigma(A^D))$ of any self-adjoint realization \tilde{A} of \mathcal{A} is unitarily equivalent to A^D . In addition, we prove that the absolutely continuous part \tilde{A}^{ac} of any realization \tilde{A} is unitarily equivalent to A^D provided that the resolvent difference $(\tilde{A} - i)^{-1} - (A^D - i)^{-1}$ is compact. The abstract results are applied to elliptic differential expressions in the half-space.

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1. Introduction

Let T be a non-negative unbounded self-adjoint operator in an *infinite* dimensional separable Hilbert space \mathcal{H} . We consider the minimal Sturm–Liouville operator A generated by the differential expression

$$\mathcal{A} := -\frac{d^2}{dt^2} + T \quad (1.1)$$

in the Hilbert space $\mathfrak{H} := L^2(\mathbb{R}_+, \mathcal{H})$ of \mathcal{H} -valued square summable vector-valued functions. Following [19,20] the minimal operator $A := A_{\min}$ is introduced to be the closure of the operator A' defined by

$$A' := \mathcal{A} \upharpoonright \mathcal{D}_0, \quad \mathcal{D}_0 := \left\{ \sum_{1 \leq j \leq k} \phi_j(t) h_j : \phi_j \in W_0^{2,2}(\mathbb{R}_+), h_j \in \text{dom}(T), k \in \mathbb{N} \right\}, \quad (1.2)$$

where $W_0^{2,2}(\mathbb{R}_+) := \{\phi \in W^{2,2}(\mathbb{R}_+) : \phi(0) = \phi'(0) = 0\}$, that is, $A_{\min} := \overline{A'}$. It is easily seen that A is a closed non-negative symmetric operator in \mathcal{H} with equal deficiency indices $n_{\pm}(A) = \dim(\mathcal{H})$. The adjoint operator A^* of $A = A_{\min}$ is the maximal operator denoted by A_{\max} . Self-adjoint extensions of A (are also called self-adjoint realizations of \mathcal{A}) were investigated for the first time by M.L. Gorbachuk [19] in the case of finite intervals I . He proved that the traces of vector-functions $f \in \text{dom}(A_{\max})$ belong to the space $\mathcal{H}_{-1/4}(T)$, cf. (5.2) and, in particular, $\text{dom}(A_{\max})$ is not contained in the Sobolev space $W^{2,2}(I, \mathcal{H})$. Based on this result he constructed a boundary triplet for the operator $A_{\max} = A_{\min}^* = A^*$ in the Hilbert space $L^2(I, \mathcal{H})$ and described all self-adjoint realizations of \mathcal{A} in terms of boundary conditions. These results are similar to those for elliptic operators in domains with smooth boundaries, cf. [4,24,34], and go back to classical papers of M.I. Višik [42] and G. Grubb [23].

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