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Global existence results for Oldroyd-B fluids in exterior domains

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ABSTRACT

In this paper we consider the set of equations describing Oldroyd-B fluids in exterior domains. It is shown that these equations admit a unique, global solution defined in a certain function space provided the initial data and the coupling constant are small enough.

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1. Introduction and main result

Incompressible fluids are being described by the set of equations

$$\begin{cases} \varrho\big(u_t + (u\cdot\nabla)u\big) = \operatorname{div}\sigma + f, \\ \operatorname{div} u = 0, \end{cases} \tag{1.1}$$

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where u denotes the velocity of the fluid, σ its stress tensor, ϱ its density and f an outer force. The stress tensor σ may be decomposed as $\sigma = -pId + \tau$, where p denotes the pressure of the fluid and τ the tangential part of the stress tensor.

In case of Newtonian fluids one has

$$\tau = 2\eta D(u),$$

where η denotes the viscosity of the fluid and $D(u) = \frac{1}{2}(\nabla u + (\nabla u)^T)$ the deformation tensor.

For many type of fluids it is impossible to describe the tangential part $\tau(t)$ of the stress tensor $\sigma(t)$ at time t knowing only $D(u(t))$. One has hence to take into account in addition the history of $D(u)$. In this case, one says that the fluid has a “memory” and is of viscoelastic type. The Oldroyd model, see [17], is one of the classical models of viscoelastic fluids and here τ is being described by the differential equation

$$\tau_t + (u \cdot \nabla)\tau + b\tau + F(\tau, \nabla\tau) = 0,$$

where $b > 0$ and F is a quadratic form in $(\tau, \nabla u)$.

In this paper, we do not consider the general Oldroyd model with eight constants but the simpler case of so-called Oldroyd-B fluids. Here τ is being determined by the equation

$$\tau + \lambda_1 \frac{D_a \tau}{Dt} = 2\eta\left[D(u) + \lambda_2 \frac{D_a D(u)}{Dt}\right], \tag{1.2}$$

where $\frac{D_a}{Dt}$ denotes the “objective derivative” given by

$$\frac{D_a \tau}{Dt} = \tau_t + (v \cdot \nabla)\tau + g_a(\tau, \nabla u)$$

and g_a is given by

$$g_a(\tau, \nabla u) = \tau W(u) - W(u)\tau - a\big[D(u)\tau + \tau D(u)\big]$$

for some $a \in [-1, 1]$. Here $W(u) = \frac{1}{2}(\nabla u - (\nabla u)^T)$ denotes the vorticity tensor; the parameters $\lambda_1, \lambda_2 \geqslant 0$ denote the relaxation and retardation time, respectively, and satisfy $\lambda_2 \leqslant \lambda_1$. Fluids of this type have viscous as well as elastic properties. Note that the case $\lambda_2 = \lambda_1 = 0$ corresponds to purely viscous fluids (being described by the Navier–Stokes equation), whereas the case $\lambda_1 > \lambda_2 = 0$ describes a purely elastic fluid.

Setting $\tau = \tau_N + \tau_E$ with

$$\tau_N = 2\eta \frac{\lambda_2}{\lambda_1} D(u),$$

it follows that τ_E satisfies

$$\tau_E + \lambda_1 \frac{D_a \tau_E}{Dt} = 2\eta\left(1 - \frac{\lambda_2}{\lambda_1}\right) D(u).$$

Setting with some abuse of notation $\tau = \tau_E$, the above set of Eqs. (1.1) and (1.2) may be rewritten in the form

$$\begin{cases} \varrho\big(u_t + (u \cdot \nabla)u\big) - \eta(1-\alpha)\Delta u + \nabla p = \operatorname{div}\tau + f, \\ \operatorname{div} u = 0, \\ \tau + \lambda_1\big(\tau_t + (u \cdot \nabla)\tau + g_a(\tau, \nabla u)\big) = 2\eta\alpha D(u), \end{cases} \tag{1.3}$$

where $\alpha = 1 - \lambda_2/\lambda_1$.

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