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Global existence and nonexistence for some degenerate and quasilinear parabolic systems

Shaohua Chen

Department of Mathematics, Physics and Geology, Cape Breton University, Sydney, Nova Scotia, B1P 6L2 Canada

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Abstract

The author discusses the degenerate and quasilinear parabolic system

$$u_t = u^\alpha v^\beta \Delta u + au^p v^q \quad \text{and} \quad v_t = u^\theta v^\eta \Delta v + bu^r v^s,$$

with Dirichlet boundary conditions in a bounded domain Ω and shows that the global existence depends crucially on the sign of the difference $(q-\beta)(r-\theta)-(\alpha+1-p)(\eta+1-s)$ and the domain Ω.

Keywords: Degenerate parabolic equation; Global existence; Global nonexistence

1. Introduction

We consider positive solutions of the following degenerate and quasilinear parabolic system:

$$\begin{cases} u_t = u^\alpha v^\beta \Delta u + au^p v^q, & \\ v_t = u^\theta v^\eta \Delta v + bu^r v^s, & t>0,\ x\in\Omega, \\ u(x,0)=\phi_1(x), \quad v(x,0)=\phi_2(x), & x\in\Omega, \\ u(x,t)=v(x,t)=0, & t>0,\ x\in\partial\Omega, \end{cases} \tag{1.1}$$

where $\alpha,\ \beta,\ p,\ q,\ \theta,\ \eta,\ r,\ s,\ a$ and b are nonnegative and $\Omega\subset R^n$ is a bounded domain with smooth boundary $\partial\Omega$. Problem (1.1) describes the processes of heat diffusion and combustion in two-component continua with complicated heat conductance and volume energy release

E-mail address: george_chen@cbu.ca.

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(see [12]). In recent years, many important results have been reported on global and blowup solutions for degenerate parabolic systems. Some of those results are stated below.

Wang [14] discussed the system

$$u_t = u^p(\Delta u + av) \quad \text{and} \quad v_t = v^q(\Delta v + bu), \tag{1.2}$$

and proved that all solutions exist globally if and only if $ab \leqslant \lambda_1^2$, where λ_1 is the first eigenvalue of $-\Delta$ in Ω with Dirichlet data. Deng, Li and Xie [7] used more general functions $f_1(u)$ and $f_2(v)$ to replace u^p and v^q, respectively, and obtained the same results. Li and Xie [11] investigated a system of n equations

$$u_{lt} = c_l u_l^{\alpha_l}\left(\Delta u_l + \prod_{j=1}^{n} u_j^{p_{lj}}\right), \quad l = 1, 2, \ldots, n,$$

and obtained very interesting results. Later, Deng [6] dealt with a general system

$$u_t = \Delta u^m + u^\alpha v^p \quad \text{and} \quad v_t = \Delta v^n + u^q v^\beta, \tag{1.3}$$

and showed that if $m > \alpha, n > \beta$ and $pq < (m-\alpha)(n-\beta)$ every nonnegative solution is global, whereas if $m < \alpha$ or $n < \beta$ or $pq > (m-\alpha)(n-\beta)$ both global and blowup solutions exist. In critical case $pq = (m-\alpha)(n-\beta)$, there exists $\lambda^* \geqslant 1$ such that all solutions are global if $\lambda_1 > \lambda^*$ and all solutions blow up in finite time if $\lambda_1 < 1/\lambda^*$. Mu et al. [12] considered the same problems and obtained similar results. Chen [3] considered the system (1.2) with lower order terms $f(u, v, Du)$ and $g(u, v, Dv)$ and showed that all solutions are bounded if $(1+c_1)\sqrt{ab} < \lambda_1$ and blow up in a finite time if $(1+c_1)\sqrt{ab} > \lambda_1$, where $c_1 > -1$ related to f and g. Chen [2] and Chen and Yu [4] also discussed single equations with lower order terms. Li et al. [9] investigated the following strong coupled system

$$u_t = v^p(\Delta u + au) \quad \text{and} \quad v_t = u^q(\Delta v + bv), \tag{1.4}$$

and proved that all solutions are global iff $\lambda_1 \geqslant \min\{a, b\}$. Other interesting results can be found in [5,10] and [13].

In this paper, we use a new method to obtain lower and upper bounds for the solutions of the regularized equations to (1.1). In this method, we estimate the integral of a ratio of one solution to the other. This method shows successful in proving existence and blowup problems (see [1–4]). Then we use the method introduced by Li et al. [9] to obtain a classical solution to (1.1).

This paper is organized as follows: In Section 2, we list some preliminary results and obtain the global existence for any smooth initial values when $(q-\beta)(r-\theta) < (\alpha+1-p)(\eta+1-s)$ plus other conditions on the indices. In Section 3, we show that the global solutions do not exist for some large initial values when $(q-\beta)(r-\theta) > (\alpha+1-p)(\eta+1-s)$ plus other conditions on the indices. In critical case $(q-\beta)(r-\theta) = (\alpha+1-p)(\eta+1-s)$, the size of the domain determines the existence of global solutions. In particular, if $q-\beta = \alpha+1-p$ and $r-\theta = \eta+1-s$, then the solutions are global for all initial values when $\lambda_1 > \lambda^*$ and the global solutions do not exist for some large initial values when $\lambda_1 < \lambda^*$, where $\lambda^* = a(b/a)^{(q-\beta)/(\eta+1-s+q-\beta)}$. We also discuss some degenerate cases. The results in this paper generalize those in [6,12,14] and [9].