Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 245 (2008) 130–153
Journal of Differential Equations
www.elsevier.com/locate/jde

Analysis of a multistate control problem related to food technology ☆

Lino J. Alvarez-Vázquez [a,*], Francisco J. Fernández [b], Rafael Muñoz-Sola [b]

[a] *Departamento de Matemática Aplicada II, ETSI Telecomunicación, Universidad de Vigo, 36310 Vigo, Spain*
[b] *Departamento de Matemática Aplicada, Facultad de Matemáticas, Universidad de Santiago de Compostela, 15706 Santiago, Spain*

Received 15 June 2007; revised 26 March 2008

Available online 28 April 2008

Abstract

This paper is concerned with an optimal control problem related to the determination of an optimal profile for the steam temperature into the autoclave along the processing of canned foods. The problem studies a system coupling the evolution Navier–Stokes equations with the heat transfer equation by natural convection (the so-called Boussinesq equations), and with the microorganisms removal equation. The essential difficulties in the study of this multistate control problem arise from the lack of uniqueness for the solution of the state system. Here we obtain—after a careful analysis of the problem mathematical formulation—the uniqueness of part of the state, and the existence of optimal solutions.

MSC: 35B37; 49J20; 93C20

Keywords: Optimal control; Multistate; Uniqueness; Existence; Food technology

☆ The research contained in this work was supported by Project MTM2006-01177 of Ministerio de Educación y Ciencia (Spain).

* Corresponding author. Fax: +34 986 812116.

E-mail addresses: lino@dma.uvigo.es (L.J. Alvarez-Vázquez), franfdz@usc.es (F.J. Fernández), rafams@usc.es (R. Muñoz-Sola).

0022-0396/$ – see front matter
doi:10.1016/j.jde.2008.03.022

1. Introduction

The most usual technique of thermal processing for long time food conservation is canning, in which the foods are previously packed and then sterilized. With this method, the containers (cans, bottles, pouches...) are heated inside an autoclave with steam during a time interval, long enough to reduce the pathogenous microorganisms concentrations down to suitable levels. Unfortunately, this technique usually makes the foods to be thermally overprocessed, because of the preventive measures usually observed by food industry. This excessive security factor in thermal processing leads to an unnecessary increase in energy costs, a nutrient degradation and a deterioration of the organoleptic properties: color, taste, smell... Mathematical modelling and optimal control, applied to this industrial problem, can offer a very interesting tool for analyzing these processes, increasing their efficiency and improving the technical design of the related equipment.

According to the heat transfer mechanism, canned foods are divided into conduction heated and convection heated. Most of the mathematical literature in this area has been devoted to conduction heated products. However, this analysis is only suitable for heating of "solid" foods, and not for "liquid" foods, where the heat transfer inside the containers occurs by natural convection. Natural convection in enclosures has been analyzed for such diverse applications as nuclear reactors, petrochemical industries or alimentary processes. In the mathematical literature related to food industry so much finite elements as finite differences methods have been applied with different rates of success in the numerical resolution of the model (cf., for instance, Stevens [15], Engelman and Sani [9], Datta [8], Kumar et al. [12], and the references therein). From a theoretical point of view, the first study on the modelling and control of sterilization processes where heat transfer occurs by natural convection has been developed by Alvarez-Vázquez and Martínez [3]. (A more complex theory for "viscous" canned foods, with temperature-dependent viscosity has been recently derived by Alvarez-Vázquez et al. in [2].)

In all of these works an artificial term involving turbulence minimization is included in the cost function, following an idea of Abergel and Temam [1] and Casas [7], in such a way that the solution of the state system be unique and, consequently, the relation control-state be single-valued. Therefore, the derivation of the optimality conditions for the optimal solutions will be an obvious (although laborious) computation. Nevertheless, for the sake of reliability, in our problem we will not take into account this turbulence term. So, we cannot expect uniqueness for the solution of the state system (since the weak solutions of the evolution three-dimensional Navier–Stokes equations are involved). Thus, we will have to deal with a so-called multistate optimal control problem, where the existence of optimal controls is not a direct result. In recent times several results have been achieved for non-well-posed optimal control problems related to the Boussinesq equations (see, for instance, Wang [18] or Li [14]), but all of them under very strong and non-realistic regularity assumptions on the velocity of the type $\vec{y} \in L^2(0,T;H^2(\Omega)^3) \cap W^{1,2}(0,T;L^2(\Omega)^3)$.

In this paper we study, from a mathematical point of view, an optimal control problem related to the determination of a suitable profile for the steam temperature in the autoclaves during the processing time. The problem deals with a system of partial differential equations coupling the Navier–Stokes equations with the heat transfer equation by natural convection (the so-called Boussinesq system), coupled with the convection–reaction–diffusion equation for the evolution of the microorganisms concentration. This problem will be subject to constraints on the control (steam temperature) and on the state (microorganisms concentration) which the process must satisfy.

Thus, in Section 2 we deeply analyze the problem: we present a rigorous mathematical formulation of the industrial problem as a boundary optimal control problem with control and state