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Global solutions for initial–boundary value problem of quasilinear wave equations

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Abstract

This work investigates the existence of globally Lipschitz continuous solutions to a class of initial–boundary value problem of quasilinear wave equations. Applying the Lax's method and generalized Glimm's method, we construct the approximate solutions of initial–boundary Riemann problem near the boundary layer and perturbed Riemann problem away from the boundary layer. By showing the weak convergence of residuals for the approximate solutions, we establish the global existence for the derivatives of solutions and obtain the existence of global Lipschitz continuous solutions of the problem.
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1. Introduction

The purpose of this work is to study the existence of globally Lipschitz continuous solutions of the following initial–boundary problem of quasilinear wave equations:

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$$\begin{cases} u_{tt} - \big(P(\rho(x), u_x)\big)_x = \rho(x) h\big(\rho(x), u, u_x\big), \\ u(x,0) = u_0(x), \quad u_t(x,0) = w_0(x), \\ u_x(0,t) = v_B(t), \end{cases} \tag{1.1}$$

where $(x,t) \in [0,\infty) \times [0,\infty)$, $u = u(x,t)$, $u_0(x)$, $w_0(x) \in \mathbb{R}$, $\rho(\cdot) : \mathbb{R} \to \mathbb{R}$ is a given continuous function with compact support, $P(\cdot) : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}^+$ and $h(\cdot) : \mathbb{R}^3 \mapsto \mathbb{R}$ are bounded smooth functions. One typical example of (1.1) arises from the application of finite elastic theory to the deformation of rubbery materials (cf. [2]) described by

$$R_{tt} - \big(p(R_r)\big)_r = g(r, R, R_r),$$

where $r := |(x,y,z)|$ denotes the variable of distance from 0 to point $(x,y,z) \in \mathbb{R}^3$ and $R = R(r,t)$ is the deformation of material, also $p(R_r) = -(R_r)^{-3}/d$ for some constant $d > 0$ and $g(r, R, R_r) = k(r)(g_1(R) + g_2(k(r), R_r))$ for some smooth functions k, g_1 and g_2.

To study the problem, we rewrite Eqs. (1.1) by

$$\begin{cases} v_t - w_x = 0, \\ w_t - \big(P\big(\rho(x), v\big)\big)_x = \rho(x) h\big(\rho(x), u, v\big), \\ w(x,0) = w_0(x), \quad v(0,t) = v_B(t), \quad u(x,0) = u_0(x), \end{cases} \tag{1.2}$$

where $v = u_x$ and $w = u_t$. Since $u(x,t) = u_0(x) + \int_0^t w(x,s)\,ds$, the above system is a differential–integral system and the problem for the existence of solutions becomes more difficult. In order to solve system (1.2) by using the methods of hyperbolic system, we give the following assumptions:

- (A_1) $\frac{\partial P}{\partial v}(\rho, v) > 0$ and $\frac{\partial^2 P}{\partial v^2}(\rho, v) < 0$ for all $\rho, v \in \mathbb{R}$;
- (A_2) there exist a continuously differentiable function $a(x)$ and a smooth function q of a such that $\rho(x) = a'(x) = q(a(x))$ for all x belong to the interior of support of ρ;
- (A_3) $u_0'(x)$, $w_0(x)$ and $v_B(t)$ belong to $L^\infty([0,\infty)) \cap \mathrm{B.V.}([0,\infty))$.

According to the assumption (A_2) and following the ideas of LeFloch [26] and Isaacson and Temple [18], we augment Eqs. (1.2) by adding the equation $a_t = 0$ and consider the following equivalent system of balance laws:

$$\begin{cases} a_t = 0, & a(x,0) = a_0(x), & \\ v_t - w_x = 0, & v(0,t) = v_B(t), & v(x,0) = u_0'(x), \\ w_t - f(a,v)_x = g(a,u,v), & w(x,0) = w_0(x), & u(x,0) = u_0(x), \end{cases} \tag{1.3}$$

where $f(a,v) = P(q(a), v)$, $g(a,u,v) = q(a) h(q(a), u, v)$, or vector form

$$U_t + F(U)_x = G(u, U), \tag{1.4}$$

with $U = (a, v, w)^T$, $F(U) = (0, -w, -f(a,v))^T$ and $G(u,U) = (0, 0, g(a,u,v))^T$.

First, we mention some of the earlier results on the subject for the homogeneous case, that is $G(u,U) \equiv 0$. The existence of weak solutions for Riemann problem was first studied by Lax [23,24,29,36]. For Cauchy problem, the existence of weak solutions was first established by

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