J. Differential Equations 250 (2011) 3859–3873

Contents lists available at ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

On dispersive effect of the Coriolis force for the stationary Navier–Stokes equations

Paweł Konieczny *, Tsuyoshi Yoneda [1]

Institute for Mathematics and Its Applications, University of Minnesota, 114 Lind Hall, 207 Church St. SE, Minneapolis, MN 55455, USA

ARTICLE INFO

Article history:
Received 27 August 2010
Revised 30 December 2010
Available online 14 January 2011

MSC:
35Q30
75D05

Keywords:
Coriolis force
Stationary Navier–Stokes equations
Dispersive effect
Large data
Fourier transform
Besov spaces

ABSTRACT

The dispersive effect of the Coriolis force for the stationary and non-stationary Navier–Stokes equations is investigated. Existence of a unique solution is shown for arbitrary large external force provided the Coriolis force is large enough. In addition to the stationary case, counterparts of several classical results for the non-stationary Navier–Stokes problem have been proven. The analysis is carried out in a new framework of the Fourier–Besov spaces.

1. Introduction

We consider the initial value problem for the 3D-Navier–Stokes equations with the Coriolis force:

$$u_t + (u \cdot \nabla)u + \Omega e_3 \times u - \Delta u + \nabla p = 0, \tag{1.1}$$

$$\nabla \cdot u = 0, \tag{1.2}$$

$$u(0, x) = u_0(x), \tag{1.3}$$

* Corresponding author.
E-mail addresses: konieczny@ima.umn.edu (P. Konieczny), yoneda@ima.umn.edu (T. Yoneda).
[1] Current address: Department of Mathematics and Statistics, University of Victoria, PO Box 3060 STN CSC, Victoria, BC, Canada, V8W 3R4.

0022-0396/$ – see front matter
doi:10.1016/j.jde.2011.01.003

where $u = u(t,x) = (u^1(t,x), u^2(t,x), u^3(t,x))$ is the unknown velocity vector field and $p = p(t,x)$ is the unknown scalar pressure at the point $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ and time $t > 0$, and $u_0(x)$ is the initial velocity vector field. Here $\Omega \in \mathbb{R}$ is the Coriolis parameter, which is the doubled angular velocity of the rotation around the vertical unit vector $e_3 = (0,0,1)$. Moreover the kinematic viscosity coefficient is normalized by one. By "$\times$" we denote the exterior product, and hence, the Coriolis term is represented by $e_3 \times u = Ju$ with the corresponding skew-symmetric 3×3 matrix J.

Problems concerning large-scale atmospheric and oceanic flows are known to be dominated by rotational effects. By this reason, almost all of the models of oceanography and meteorology dealing with large-scale phenomena include a Coriolis force. For example, an oceanic circulation featuring a hurricane is caused by the large rotation. There is no doubt that other physical effects are of similar significance like salinity, natural boundary conditions and so on. However, the first step in the study of more complex model is to understand the behavior of rotating fluids. To this end, we treat in a standard manner the Navier–Stokes equations with the Coriolis force.

Let us look back on the history of the Coriolis force. In 1868, Kelvin observed that a sphere moving along the axis of uniformly rotating water takes with it a column of liquid as if this were a rigid mass (see [9] for references). After that, Hough [16], Taylor [23] and Proudmann [22] made important contributions. Mathematically, Poincaré [22], and more recently, Babin et al. [1,2] considered non-stationary Navier–Stokes equations with Coriolis force in periodic case. The periodicity is extended to the almost periodic case by several authors. In particular for the results on the local existence of solutions to the non-stationary rotating Navier–Stokes equations with spatially almost periodic data and its properties, we refer the reader to [10,13,14]. Moreover, for the results on the global in time existence of solutions and their long time behavior in the almost periodic setting, see [11,12,25] for example.

On the other hand, Chemin et al. [7] considered the decaying data case. They derived dispersion estimates on a linearized version of the 3D-Navier–Stokes equations with the Coriolis force to show the existence of a global in time solution to the non-stationary rotating Navier–Stokes system. To construct such estimate, they handled eigenvalues and eigenfunctions of the Coriolis operator.

The main result of this paper is to show existence of the solution to the stationary Navier–Stokes equations with the Coriolis force for arbitrary large external force provided that the Coriolis force is sufficiently large (compare it with results for the Navier–Stokes equations (2.8) with $\Omega = 0$, for example [18] for the case of exterior domain). To do so, we introduce a new type of function spaces, namely, Fourier–Besov spaces (FB), which allow us to describe, in a clear way, how the Coriolis force influences solutions to the considered system. A similar approach, based on function spaces which make the analysis of specific features of a system much easier, has been used by the first author in [17], where an asymptotic structure of solution to the stationary Navier–Stokes equations in $\mathbb{R}^2$ was investigated.

In FB spaces, we cannot expect energy type estimates and a structure of Hilbert spaces as in [7]. The main motivation to introduce those spaces is that, in this framework, we are able to present directly dispersive effect of the Coriolis force (see Proposition 2.5), which is in principle different from the dispersive effect from [7] (for the space-time estimates for different equations see for example [19,20]).

To emphasize the usefulness of introduced spaces we prove the existence of solutions to the non-stationary Navier–Stokes–Coriolis system in function spaces which are counterparts for well-known classical results in the Navier–Stokes theory (see [3,5,6]). Moreover, we can considerably simplify other results for the Navier–Stokes–Coriolis system, like recent results by Giga et al. [12].

2. Main results

In the paper we follow basic ideas from the Littlewood–Paley theory. We denote by $\varphi \in \mathcal{S}(\mathbb{R}^3)$ a radially symmetric function supported in $\{\xi \in \mathbb{R}^3 \colon \frac{3}{4} \leqslant |\xi| \leqslant \frac{8}{3}\}$ such that

$$\sum_{j \in \mathbb{Z}} \varphi\big(2^{-j}\xi\big) = 1 \quad \text{for all } \xi \neq 0.$$