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Pointwise Green function bounds and stability of combustion waves ☆

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Abstract

Generalizing similar results for viscous shock and relaxation waves, we establish sharp pointwise Green function bounds and linearized and nonlinear stability for traveling wave solutions of an abstract viscous combustion model including both Majda's model and the full reacting compressible Navier–Stokes equations with artificial viscosity with general multi-species reaction and reaction-dependent equation of state, under the necessary conditions of strong spectral stability, i.e., stable point spectrum of the linearized operator about the wave, transversality of the profile as a connection in the traveling-wave ODE, and hyperbolic stability of the associated Chapman–Jouguet (square-wave) approximation. Notably, our results apply to combustion waves of any type: weak or strong, detonations or deflagrations, reducing the study of stability to verification of a readily numerically checkable Evans function condition. Together with spectral results of Lyng and Zumbrun, this gives immediately stability of small-amplitude strong detonations in the small heat-release (i.e., fluid-dynamical) limit, simplifying and greatly extending previous results obtained by energy methods by Liu–Ying and Tesei–Tan for Majda's model and the reactive Navier–Stokes equations, respectively.

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1. Introduction

In this paper, we extend the viscous shock stability theory of [17,35–37,50] to traveling waves of combustion models, including the simplified combustion model of Majda, and an artificial viscosity version of the reacting Navier–Stokes equations. Specifically, we (i) derive sharp pointwise Green function bounds, yielding a sharp $L^1 \cap L^p \to L^p$ linearized stability criterion in terms of an Evans function condition, and (ii) assuming the Evans stability condition, establish nonlinear stability for waves of arbitrary type: weak or strong detonation, weak of strong deflagration.

This reduces the question of linear and nonlinear stability to verification of the Evans condition, an ODE criterion that is readily checked numerically [3–6,20]. It opens also the possibility of analytic verification, in particular in various asymptotic limits. For example, the Evans condition has already been verified analytically for small-amplitude strong detonations in the small heat-release limit [28] and for arbitrary-amplitude strong detonations of Majda's model [40], so that our theory yields complete nonlinear stability results in these cases, recovering and greatly extending previous results of [25,27,44]. More general situations are likely to require numerical investigation, as is standard in the combustion literature even for the simpler reactive Euler (ZND) model in which viscosity and heat conduction are neglected; see, e.g., [23].

In the numerical setting, the value of our results is that Evans function calculations consist in standard and numerically well-conditioned boundary-value ODE, whereas a "direct" nonlinear stability analysis by numerical approximation of the original evolutionary PDE is numerically much more sensitive and computationally intensive. Moreover, Evans function conclusions come with a guarantee, as such numerical experiments do not; indeed, it is quite feasible, as discussed in [4], to convert a numerical Evans function analysis to a numerically-aided proof.

1.1. Combustion models

We show that viscous shock and combustion waves, like their hyperbolic counterparts, can be studied within a common framework. Indeed, viscous shocks, viscous detonations, and relaxation shocks may all be considered as traveling waves of the special class of hyperbolic–parabolic balance laws, or reaction–diffusion–convection equations,

$$U_t + \mathcal{F}(U) = 0, \qquad \mathcal{F}(U) = F(U)_x - \big(B(U)U_x\big)_x - G(U), \tag{1.1}$$

having the damping property

$$\Re\sigma(dG) \leqslant 0, \tag{1.2}$$

where (here and elsewhere) $\sigma(M)$ denotes spectrum of a matrix or linear operator M. For viscous shocks, $G \equiv 0$, while for relaxation shocks, dG has constant rank, its kernel corresponding to a local equilibrium manifold.

By contrast, combustion equations have the composite structure

$$G(U) = \phi(U)\tilde{G}(U),$$

where ϕ is a scalar "ignition function" that turns the reaction on or off—specifically, it is zero on some subset of the state space and positive elsewhere—and $\tilde{G}$ is a relaxation type term, $d\tilde{G}$