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ScienceDirect
J. Differential Equations 228 (2006) 580–610
Journal of Differential Equations
www.elsevier.com/locate/jde

Global attractivity of positive periodic solution to periodic Lotka–Volterra competition systems with pure delay ☆

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Received 2 August 2005

Abstract

We consider a *periodic* Lotka–Volterra competition system without instantaneous negative feedbacks (i.e., pure-delay systems)

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j\big(t - \tau_{ij}(t)\big)\right], \quad i = 1, 2, \ldots, n. \tag{$*$}$$

We establish some 3/2-type criteria for global attractivity of a positive periodic solution of the system, which generalize the well-known Wright's 3/2 criteria for the autonomous delay logistic equation, and thereby, address the open problem proposed by both Kuang [Y. Kuang, Global stability in delayed nonautonomous Lotka–Volterra type systems without saturated equilibria, Differential Integral Equations 9 (1996) 557–567] and Teng [Z. Teng, Nonautonomous Lotka–Volterra systems with delays, J. Differential Equations 179 (2002) 538–561].

MSC: primary 34K13; secondary 34K20, 92D25

☆ Partially supported by NNSF of China (No. 10471153) (X.T., D.C.), by NSERC of Canada and PREA of Ontario.
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[1] This paper was finalized when X.T. was visiting AMSS, Chinese Academy of Sciences.
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0022-0396/$ – see front matter
doi:10.1016/j.jde.2006.06.007

Keywords: Positive periodic solution; Lotka–Volterra competition system; Pure delay; 3/2-type conditions; Global attractivity

1. Introduction

One of the most celebrated models for population dynamics is the Lotka–Volterra system. Due to its theoretical and practical significance, the Lotka–Volterra system has been extensively and intensively studied (see, e.g., [1–55]). In order to reflect the seasonal fluctuations, it is reasonable to study the Lotka–Volterra system with *periodic coefficients*. A very basic and important ecological problem associated with the study of multi-species population interaction in a periodic environment is the existence and global attractivity of a positive periodic solution. Such a problem also arises in many other contexts.

In this paper, we investigate the following *periodic n*-species Lotka–Volterra competition system with delays

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j\big(t-\tau_{ij}(t)\big)\right], \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where $r_i, a_{ij}, \tau_{ij} \in C(\mathbb{R},\ \mathbb{R}^+ = [0,\infty))$ are ω-periodic functions ($\omega > 0$) with

$$\bar{r}_i = \frac{1}{\omega}\int_0^{\omega} r_i(s)\,ds > 0, \quad \bar{a}_{ij} = \frac{1}{\omega}\int_0^{\omega} a_{ij}(s)\,ds \geqslant 0, \quad i, j = 1, 2, \ldots, n. \tag{1.2}$$

The existence and attractivity of the positive periodic solutions of some special cases of Eq. (1.1) have been studied extensively. Many important results can be found in [1–19,22–32,34–55] and references cited therein. In those works the method of Liapunov functions [5,21], the theory of monotone semiflows generated by functional differential equations [39,40], the fixed point theory [9], and so on are extensively applied. Recently, the un-delayed version of (1.1), i.e., system (1.1) with $\tau_{ij}(t) \equiv 0$, $i, j = 1, 2, \ldots, n$, was studied by Redheffer [34,35] and Tineo [51]. Under remarkably weak conditions (see [34, conditions (a)–(e)] and [51, condition (0.2)]), the boundedness, permanence, global attractivity, and existence of positive periodic solutions are obtained (see [34, Theorem 1], [51, Sections 3 and 4]). Recently, Teng [47] extended the main results in Redheffer [34] to the following delayed system

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - a_{ii}(t)x_i(t) - \sum_{j\neq i}^{n} a_{ij}(t)x_j\big(t-\tau_{ij}(t)\big)\right], \quad i = 1, 2, \ldots, n, \tag{1.3}$$

which is a special form of system (1.1) with $\tau_{ii}(t) \equiv 0$, $i = 1, 2, \ldots, n$. In [47], the author fully took advantage of the fact that there is no delay in the negative feedback terms $a_{ii}(t)x_i(t)$, $i = 1, 2, \ldots, n$ (i.e., the system has instantaneous negative feedbacks). Therefore, the methods used in [47] would fail when applying to the pure-delay system (1.1), due to the lack of the instantaneous negative feedbacks.

On the *existence* of positive periodic solutions of system (1.1), Tang and Zou [46] recently obtained the following rather general results.