J. Differential Equations 249 (2010) 1035–1055

Contents lists available at ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde

On a class of nonlinear elliptic equations with fast increasing weight and critical growth ☆

Marcelo F. Furtado [a,*], Olímpio H. Myiagaki [b], João Pablo P. da Silva [c]

[a] Departamento de Matemática, Universidade de Brasília, 70910-900 Brasília-DF, Brazil
[b] Departamento de Matemática, Universidade de Federal de Viçosa, 36570-000, Viçosa-MG, Brazil
[c] Departamento de Matemática, Universidade Federal do Pará, 66075-110, Belém-PA, Brazil

ARTICLE INFO

Article history:
Received 2 January 2010
Revised 12 April 2010
Available online 27 April 2010

Keywords:
Critical problems
Self-similar solutions
Positive solutions
Nodal solutions

ABSTRACT

We are concerned with the existence of rapidly decaying solutions for the equation

$$-\mathrm{div}\big(K(x)\nabla u\big) = \lambda K(x)|x|^{\beta}|u|^{q-2}u + K(x)|u|^{2^*-2}u, \quad x \in \mathbb{R}^N,$$

where $N \geqslant 3$, $2 \leqslant q < 2^* := 2N/(N-2)$, $\lambda > 0$ is a parameter, $K(x) := \exp(|x|^{\alpha}/4)$, $\alpha \geqslant 2$ and the number β is given by $\beta := (\alpha-2)\frac{(2^*-q)}{(2^*-2)}$. We obtain a positive solution if $2 < q < 2^*$ and a sign changing solution if $q = 2$. The existence results depend on the values of the parameter λ. In the proofs we apply variational methods.

© 2010 Elsevier Inc. All rights reserved.

1. Introduction

We consider the nonlinear equation

$$-\,\mathrm{div}\big(K(x)\nabla u\big) = \lambda K(x)|x|^{\beta}|u|^{q-2}u + K(x)|u|^{2^*-2}u, \quad x \in \mathbb{R}^N, \tag{P}$$

where $N \geqslant 3$, $2 \leqslant q < 2^* := 2N/(N-2)$, $\lambda > 0$ is a parameter, $K(x) := \exp(|x|^{\alpha}/4)$, $\alpha \geqslant 2$ and the number β is given by

☆ The first and second authors were partially supported by CNPq/Brazil.
* Corresponding author.
E-mail addresses: mfurtado@unb.br (M.F. Furtado), olimpio@ufv.br, ohmiyagaki@gmail.com (O.H. Myiagaki), jpablo_ufpa@yahoo.com.br (J.P.P. da Silva).

0022-0396/$ – see front matter © 2010 Elsevier Inc. All rights reserved.
doi:10.1016/j.jde.2010.04.014

$$\beta := (\alpha - 2)\frac{(2^* - q)}{(2^* - 2)}.$$

As pointed out in [7], one of the motivations for studying the above equation relies on the fact that, for $\alpha = q = 2$ and $\lambda = (N-2)/(N+2)$, it appears when one tries to find self-similar solutions

$$v(x,t) = t^{\frac{2-N}{N+2}} u\left(xt^{\frac{-1}{2}}\right)$$

to the parabolic equation

$$v_t - \Delta v = |v|^{\frac{4}{N-2}} v, \quad \mathbb{R}^N \times (0, +\infty).$$

The radially symmetric case with $\alpha = q = 2$ was considered in [1]. As far as we know, the first variational approach was done by Escobedo and Kavian in [6]. In that article the authors have considered $\alpha = q = 2$ and $N \geqslant 3$, and have proved that the existence of positive solutions is related with the interaction of the parameter λ with the first positive eigenvalue of the associated linear problem

$$-\operatorname{div}\left(K(x)\nabla u\right) = \lambda K(x)|x|^{\alpha-2}u, \quad x \in \mathbb{R}^N. \tag{LP}$$

Among other results, they have noticed a dichotomy in the existence range of λ for $N = 3$, relative to space dimensions $N \geqslant 4$. More precisely, for $N \geqslant 4$, there is a solution if and only if $\lambda \in (N/4, N/2)$. If $N = 3$, there is a positive solution for $\lambda \in (1, 3/2)$, and there is no solution for $\lambda \leqslant 3/4$ and $\lambda \geqslant 3/2$.

Later on, many authors considered the case $\alpha = q = 2$ and addressed questions of existence, symmetry and asymptotic behavior of solutions of (P), of the associated parabolic equation and its variants (see [8,10,12,9] and references therein). We also quote the paper of Ohya [11], where some results for a p-Laplacian type operator can be found.

Recently, Catrina, Furtado and Montenegro [4] have obtained some results for $\alpha \geqslant 2$ and $q = 2$. After calculating the first eigenvalue of (LP) as $\lambda_1 = \alpha(N - 2 + \alpha)/4$, they have proved that, if $2 \leqslant \alpha \leqslant N - 2$, then the problem (P) has a positive solution if, and only if, $\lambda \in (\lambda_1/2, \lambda_1)$. If $\alpha > N - 2$ and $\lambda \in (\alpha^2/4, \lambda_1)$ then the problem (P) has a positive solution. Moreover, also in this last case, the problem has no solution if $\lambda \leqslant \lambda_1/2$ or $\lambda \geqslant \lambda_1$. So, if $\alpha > 2$, the critical dimension of the problem depends on the value of α.

Due to the presence of the critical Sobolev exponent in (P), it is natural to make a parallel with the Brezis and Nirenberg problem

$$-\Delta u = \lambda |u|^{q-2}u + |u|^{2^*-2}u, \quad u \in H_0^1(\Omega), \tag{BN}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain and $N \geqslant 3$. The aforementioned results of [6,4] can be viewed as versions of those ones presented in [2] for the above problem when $q = 2$. The nonexistence results for $\lambda \geqslant \lambda_1$ are a consequence of a Pohozaev type identity. In the case that $2 < q < 2^*$, this identity does not give any information. So, we can expect existence of solution for any $\lambda > 0$. A result in this direction to the problem (BN) was presented in [2, Section 2]. In our first result we give an answer for this question when we deal with the problem (P). More specifically, we shall prove the following result.

Theorem 1.1. *The problem (P) has a positive solution in each of the following cases*

(i) $N \geqslant \alpha + 2$, $2 < q < 2^*$, $\lambda > 0$;
(ii) $2 < N < \alpha + 2$, $2^* - \frac{4}{\alpha} < q < 2^*$, $\lambda > 0$;
(iii) $2 < N < \alpha + 2$, $2 < q \leqslant 2^* - \frac{4}{\alpha}$, $\lambda > 0$ *is sufficiently large.*

Download English Version:

https://daneshyari.com/en/article/4612466

Download Persian Version:

https://daneshyari.com/article/4612466

Daneshyari.com