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Existence and controllability results for nondensely defined impulsive semilinear functional differential inclusions

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ABSTRACT

In this paper, we shall establish sufficient conditions for the existence of integral solutions and extremal integral solutions for some nondensely defined impulsive semilinear functional differential inclusions in separable Banach spaces. We shall rely on a fixed point theorem for the sum of completely continuous and contraction operators. The question of controllability of these equations and the topological structure of the solutions set are considered too.

1. Introduction

In this paper, we shall be concerned with existence of integral solutions and extremal integral solutions defined on a compact real interval for first-order impulsive semilinear functional inclusions in

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a separable Banach space. In Section 3, we will consider the following first-order impulsive semilinear differential inclusions of the form

$$y'(t) - Ay(t) \in F(t, y_t), \quad \text{a.e. } t \in J = [0, b],\ t \neq t_k,\ k = 1, \ldots, m, \tag{1}$$

$$\Delta y|_{t=t_k} \in I_k\big(y\big(t_k^-\big)\big), \quad k = 1, \ldots, m, \tag{2}$$

$$y(t) = \phi(t), \quad t \in [-r, 0], \tag{3}$$

where $F : J \times D \to \mathcal{P}(E)$ is a given function, $D = \{\overline{\psi} : [-r, 0] \to E,\ \psi$ is continuous everywhere except for a finite number of points s at which $\overline{\psi}(s^-), \overline{\psi}(s^+)$ exist and $\overline{\psi}(s^-) = \overline{\psi}(s)\}$, $\phi \in D$ $(0 < r < \infty)$, $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = b$, $A : D(A) \subset E \to E$ is a nondensely defined closed linear operator on E, $I_k : E \to \mathcal{P}(E)$ $(k = 1, 2, \ldots, m)$ are multivalued maps with closed graph, $y(t_k^-)$ and $y(t_k^+)$ represent the left and right limits at $t = t_k$ of $y(t)$. Finally E is a real separable Banach space equipped with the norm $|.|$ and $\mathcal{P}(E)$ denotes the family of all nonempty subsets of E. For any continuous function y defined on $[-r, b]\backslash\{t_1, t_2, \ldots, t_m\}$ and any $t \in J$, we denote by y_t the element of D defined by

$$y_t(\theta) = y(t + \theta), \quad \theta \in [-r, 0].$$

Here $y_t(\cdot)$ represents the history of the state from $t - r$, up to the present time t.

Functional differential equations arise in a variety of areas of biological, physical, and engineering applications, see, for example, the books of Hale [35], Hale and Verduyn Lunel [36], Kolmanovskii and Myshkis [42] and Wu [55], and the references therein.

Impulsive differential and partial differential equations are used to describe various models of real processes and phenomena studied in physics, chemical technology, population dynamics, biotechnology and economics. That is why in recent years they have been the object of investigations. We refer to the monographs of Bainov and Simeonov [12], Benchohra et al. [16], Lakshmikantham et al. [43], and Samoilenko and Perestyuk [52] where numerous properties of their solutions are studied, and a detailed bibliography is given. Semilinear functional differential equations and inclusions with or without impulses have been extensively studied where the operator A generates a C_0-semigroup. Existence and uniqueness, among other things, are derived; see the books of Benchohra et al. [14], Heikkila and Lakshmikantham [37], Kamenskii et al. [39] and the papers by Liu [46], Rogovchenko [49,50] and Sakthivel et al. [51]. Recently the existence of integral solution on compact intervals for the problem (1)–(3) has been considered by Benchohra et al. [15] and Ezzinbi and Liu [30]. It has been shown that the density condition is not necessary to deal with partial functional differential equations. For more details and examples on nondensely defined operators and the concept of integrated semigroup we refer to the monograph [5] and Engel and Nagel [29], and to the papers by Adimy et al. [3], Adimy and Ezzinbi [4], Benchohra et al. [15], Ezzinbi and Liu [30] and to papers [9,10,25]. Controllability of differential inclusions with different conditions has been considered in the monograph of Benchohra et al. [14] and in the papers of Balachandran and Dauer [11], Benchohra et al. [13], Benchohra and Ntouyas [17], Fu [31], and Li and Xue [45] and the references cited therein. In [2] Abada et al. have studied the controllability of a class of impulsive semilinear functional differential inclusions in Fréchet spaces by means of the extrapolation method [24,29], and in [1] the existence of mild and extremal mild solutions for first-order semilinear densely defined impulsive functional differential inclusions in separable Banach spaces with local and nonlocal conditions has been considered. To the best of our knowledge, there are very few results for impulsive evolution inclusions with multivalued jump operators; see [1,18,47]. Our goal here is to give existence results for the problem (1)–(3) in the case of nondensely defined operators and with multivalued jumps. In Section 2, we will recall briefly some basic definitions and preliminary facts which will be used throughout the following sections. In Section 3, we present some examples of nondensely defined operators. In Section 4, we prove existence of integral solutions for problem (1)–(3). Our approach will be based on a fixed point theorem due to Dhage [27] for the sum of contraction and completely continuous multivalued operators. The compactness of the solutions set of problem (1)–(3) will end this section. In Section 5,