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Long time behavior for the weakly damped driven long-wave–short-wave resonance equations ☆

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Abstract

In this paper, we consider the Cauchy problem of the long-wave–short-wave resonance equations. By making use of a Strichartz-type inequality for the solutions, decomposing suitably the solution semigroup into a decay parts and a more regular parts, and ruling out the "vanishing" and "dichotomy" of the solutions, we prove the existence of the global attractor and the asymptotic smoothing effect of the solutions.

MSC: 35Q55; 37L50

Keywords: Long-wave–short-wave resonance equations; Strichartz-type inequality; Decomposition of the solution semigroup; Asymptotic smoothing effect; Global attractor

1. Introduction

In this paper, we consider the following long-wave–short-wave resonance equations of the following form:

$$iu_t + u_{xx} - uv + i\alpha u = f(x), \quad x \in \mathbf{R}, \quad t > 0, \tag{1.1}$$

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$$v_t + \beta v + \gamma(|u|^2)_x = g(x), \quad x \in \mathbf{R}, \ t > 0 \tag{1.2}$$

associated with the initial conditions

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \ x \in \mathbf{R}, \tag{1.3}$$

where the unknown complex function u is the envelope of the short wave, and the unknown real function v is the amplitude of the long wave. The constants α, β are positive and stand for the Landau damping, and $\gamma \in \mathbf{R}\backslash\{0\}$ is real and stands for the dispersion interaction.

The long-wave–short-wave resonance equations of the above type arise in the study of the interaction of the surface waves with both gravity and capillary modes presence (see [5–7]) and also in the analysis of internal waves, as well as Rossby waves [11]. In the plasma physics they describe the resonance of the high-frequency electron plasma oscillation and the associated low-frequency ion density perturbation [22]. In the Hamiltonian case they are of the inverse scattering type and have solitary waves [21,29].

Due to their rich physical and mathematical properties the long-wave–short-wave resonance equations have drawn much attention of many physicists and mathematicians. For its Hamiltonian case and more general types, the existence of solutions, the solitary waves and their stability have been quite extensively studied. Guo [12] studied the well-posedness of solutions in the usual Sobolev spaces. Tsutsumi and Hatano [26,27] studied the well-posedness of solutions in fractional Sobolev spaces. These results are improved by Bekiranov et al. [3,4] which investigate the well-posedness of solutions with low regularity via Bourgain's method. For the solitary waves and their stability, we refer to Angulo and Montenegro [1], Guo and Chen [13], Guo and Pan [15], Laurençot [19], Pava and Montenegro [23], etc.

In this paper, we study the long time behavior of the solutions of (1.1)–(1.3). We remark that these equations have no smoothing property and the embedding between usual Sobolev spaces lacks the compactness due to the unboundedness of the spatial domain. To overcome these difficulties we make use of some Strichartz-type inequality, the decomposition technique and also an idea from the concentration compactness. We use Strichartz-type inequality and an energy equality method to show the existence and continuity of the semigroup generated by the equations. We obtain the asymptotic smoothing effect of solutions by decomposing the semigroup into a decay part and a more regular part. In passing limits as t goes to infinity we have to overcome the noncompactness of the usual Sobolev embedding. We borrow the idea from the concentration compactness principle and rule out the "vanishing" and "dichotomy" via a cut-off function.

The Strichartz-type inequalities of space–time estimates and the energy method have been successfully applied to the study of the long time behavior of solutions of dissipative partial differential equations, especially when the problems lack the compact embedding and smoothing properties, for examples, see [8–10,17,20,28], etc. In unbounded domain cases weighted spaces are used to recover the compact embeddings (see [2,14], etc.). The decomposition technique have been also applied in [9,10], where