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On asymptotic behavior of solutions to Korteweg–de Vries type equations related to vortex filament with axial flow

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Abstract

We study the global existence and asymptotic behavior in time of solutions to the Korteweg–de Vries type equation called as "Hirota" equation. This equation is a mixture of cubic nonlinear Schrödinger equation and modified Korteweg–de Vries equation. We show the unique existence of the solution for this equation which tends to the given "modified" free profile by using the two asymptotic formulae for some oscillatory integrals.

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1. Introduction

In this paper we consider the large time behavior of solutions to the Korteweg–de Vries type equations:

$$i\partial_t u + \partial_x^2 u + i\mu\partial_x^3 u = -\frac{1}{2}|u|^2 u - \frac{3}{2}i\mu|u|^2\partial_x u, \quad t, x \in \mathbb{R}, \tag{1.1}$$

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where u is a complex-valued unknown function and μ is the non-zero real constant. (1.1) is called "Hirota" equation and a mixture of cubic nonlinear Schrödinger equation and modified Korteweg–de Vries equation. Indeed, taking $\mu = 0$ in (1.1), we obtain the cubic nonlinear Schrödinger equation:

$$i\partial_t u + \partial_x^2 u = -\frac{1}{2}|u|^2 u, \quad t, x \in \mathbb{R}. \tag{1.2}$$

On the other hand, dropping $\partial_x^2 u$ and $-\frac{1}{2}|u|^2 u$ in (1.1), we obtain the complex-valued modified Korteweg–de Vries equation:

$$\partial_t u + \mu\partial_x^3 u = -\frac{3}{2}\mu|u|^2\partial_x u. \tag{1.3}$$

In this paper we show the unique existence of the solution to (1.1) which tends to the given "modified" free profiles.

Firstly, we explain the physical background of Eq. (1.1). (1.1) has been derived by Hirota Ryogo as a model of the soliton equation. This equation also arises in the study of the motion of a vortex filament. We consider the three-dimensional motion of an isolated vortex filament embedded in inviscid incompressible fluid fulfilled in an infinite region. In [2], Da Rios introduced the following model for the motion of the vortex filament by using "localized induction approximation": We denote the centerline of the vortex filament by $\mathbf{X} = \mathbf{X}(t, x)$, represented as functions of time t and arc length x. Let (κ, τ) be the curvature, torsion and let $(\mathbf{t}, \mathbf{n}, \mathbf{b})$ be the Frenet–Serret frame of the centerline of the vortex filament, respectively. Then $\mathbf{X}$ satisfies the following equation:

$$\partial_t \mathbf{X} = \kappa\mathbf{b}. \tag{1.4}$$

By introducing "the Hasimoto transform" (see [8]) defined by

$$u(t, x) = \kappa(t, x)e^{i\int_0^x \tau(t,y)\,dy}, \tag{1.5}$$

Eq. (1.4) is transformed to the cubic nonlinear Schrödinger equation (1.2). Hence, the localized induction equation (1.4) is completely integrable equation equivalent to the cubic nonlinear Schrödinger equation.

To describe the motion of actual vortex filament precisely, some detailed models taking into account of the effect from higher order corrections of equation have been introduced by several authors (see e.g., Fukumoto [3]).

Fukumoto and Miyazaki [4] proposed the following equation for the motion of a vortex filament with axial flow:

$$\partial_t \mathbf{X} = \kappa\mathbf{b} - \mu\left\{\frac{1}{2}\kappa^2\mathbf{t} + \partial_x\kappa\mathbf{n} + \kappa\tau\mathbf{b}\right\}, \tag{1.6}$$

where μ is real constant. Similar to (1.4), the above equation (1.6) is transformed by the Hasimoto map (1.5) to the Korteweg–de Vries type equation (1.1).

There exist several results on the existence of a global solution to (1.1) and (1.6). Laurey [14] (see also Staffilani [19]) proved the global well-posedness for the initial value problem of (1.1)