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Existence and stability of traveling wave fronts in reaction advection diffusion equations with nonlocal delay

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Abstract

This paper is concerned with the existence, uniqueness and globally asymptotic stability of traveling wave fronts in the quasi-monotone reaction advection diffusion equations with nonlocal delay. Under bistable assumption, we construct various pairs of super- and subsolutions and employ the comparison principle and the squeezing technique to prove that the equation has a unique nondecreasing traveling wave front (up to translation), which is monotonically increasing and globally asymptotically stable with phase shift. The influence of advection on the propagation speed is also considered. Comparing with the previous results, our results recovers and/or improves a number of existing ones. In particular, these results can be applied to a reaction advection diffusion equation with nonlocal delayed effect and a diffusion population model with distributed maturation delay, some new results are obtained.

MSC: 35R10; 35B40; 34K30; 58D25

Keywords: Existence; Uniqueness; Asymptotic stability; Traveling wave front; Reaction advection diffusion equation; Nonlocal delay; Bistable

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Contents

1. Introduction

In this paper, we are concerned with an one space dimensional reaction advection diffusion equation with nonlocal delay of the form

$$\frac{\partial u}{\partial t} = d\Delta u + B\frac{\partial u}{\partial x} + g\big(u(x,t), \big(h * S(u)\big)(x,t)\big), \quad x \in \mathbb{R},\ t > 0, \tag{1.1}$$

where $d > 0$, $B \in \mathbb{R}$, Δ is the Laplacian operator on $\mathbb{R}$, h is a nonnegative kernel satisfying

$$\int_0^\tau \int_{-\infty}^\infty h(y,s)\,dy\,ds = 1, \qquad \int_0^\tau \int_{-\infty}^\infty |y| h(y,s)\,dy\,ds < \infty, \tag{1.2}$$

and the convolution is defined by

$$\big(h * S(u)\big)(x,t) = \int_{-\tau}^0 \int_{-\infty}^\infty h(x-y,-s) S\big(u(y,t+s)\big)\,dy\,ds.$$

For $g(u,v)$ and $S(u)$, we impose the following conditions:

(H1) $g \in C^2([0,1] \times [S(0),S(1)],\mathbb{R})$ and $\partial_2 g(u,v) \geqslant 0$ for $(u,v) \in [0,1] \times [S(0),S(1)]$; $S \in C^2([0,1],\mathbb{R})$ and $S'(u) \geqslant 0$ for $u \in [0,1]$.

(H2) $g(0,S(0)) = g(1,S(1)) = 0$, $\partial_1 g(0,S(0)) + \partial_2 g(0,S(0))S'(0) < 0$, and $\partial_1 g(1,S(1)) + \partial_2 g(1,S(1))S'(1) < 0$.

Under condition (H2), it is obvious that 0 and 1 are stable equilibria of (1.1). We are interested in traveling wave solutions that connect the two stable equilibria 0 and 1. Throughout this paper, a *traveling wave solution* of (1.1) always refers to a pair (U,c), where $U = U(\xi)$ is a function on $\mathbb{R}$ and c is a constant, such that $u(x,t) := U(x-ct)$ is a solution of (1.1) and

$$\lim_{\xi \to -\infty} U(\xi) = 0, \qquad \lim_{\xi \to +\infty} U(\xi) = 1. \tag{1.3}$$

We call c the *traveling wave speed* and U the profile of the wave front. If $c = 0$, we say U is a *standing wave*. Moreover, we say a traveling wave $U(x-ct)$ is monotone if $U(\cdot):\mathbb{R} \to \mathbb{R}$ is a strictly increasing function.