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Existence theory for positive solutions to one-dimensional p-Laplacian boundary value problems on time scales

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Abstract

In this paper we consider the one-dimensional p-Laplacian boundary value problem on time scales

$$\big(\varphi_p\big(u^{\Delta}(t)\big)\big)^{\Delta} + h(t) f\big(u^{\sigma}(t)\big) = 0, \quad t \in [a, b],$$

$$u(a) - B_0\big(u^{\Delta}(a)\big) = 0, \qquad u^{\Delta}\big(\sigma(b)\big) = 0,$$

where $\varphi_p(u)$ is p-Laplacian operator, i.e., $\varphi_p(u) = |u|^{p-2}u$, $p > 1$. Some new results are obtained for the existence of at least single, twin or triple positive solutions of the above problem by using Krasnosel'skii's fixed point theorem, new fixed point theorem due to Avery and Henderson and Leggett–Williams fixed point theorem. This is probably the first time the existence of positive solutions of one-dimensional p-Laplacian boundary value problems on time scales has been studied.

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Keywords: Time scales; Positive solution; Cone; Fixed point

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1. Introduction

The study of dynamic equations on time scales goes back to its founder Stefan Hilger [24], and is a new area of still fairly theoretical exploration in mathematics. Motivating the subject is the notion that dynamic equations on time scales can build bridges between continuous and discrete mathematics. Further, the study of time scales has led to several important applications, e.g., in the study of insect population models, phytoremediation of metals, wound healing, and epidemic models [15,25,26,38].

We begin by presenting some basic definitions which can be found in [1,2,14,24,29]. Another excellent source on dynamic equations on time scales is the book [15].

A time scale $\mathbb{T}$ is a nonempty closed subset of $\mathbb{R}$. It follows that the jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$

$$\sigma(t) = \inf\{\tau \in \mathbb{T}:\ \tau > t\} \quad \text{and} \quad \rho(t) = \sup\{\tau \in \mathbb{T}:\ \tau < t\}$$

(supplemented by $\inf\emptyset := \sup\mathbb{T}$ and $\sup\emptyset := \inf\mathbb{T}$) are well defined. The point $t \in \mathbb{T}$ is left-dense, left-scattered, right-dense, right-scattered if $\rho(t) = t$, $\rho(t) < t$, $\sigma(t) = t$, $\sigma(t) > t$, respectively. If $\mathbb{T}$ has a left-scattered maximum M, define $\mathbb{T}^\kappa = \mathbb{T} - \{M\}$; otherwise, set $\mathbb{T}^\kappa = \mathbb{T}$. The forward graininess is $\mu(t) := \sigma(t) - t$.

Throughout this paper, we make the blanket assumption that $a < b$ are points in $\mathbb{T}$, and

$$[a, b] = \{t \in \mathbb{T}:\ a \leqslant t \leqslant b\}.$$

For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^\kappa$, the delta derivative of f at t, denoted by $f^\Delta(t)$, is the number (provided it exists) with the property that given any $\epsilon > 0$, there is a neighborhood $U \subset \mathbb{T}$ of t such that

$$\left| f\big(\sigma(t)\big) - f(s) - f^\Delta(t)\big[\sigma(t) - s\big]\right| \leqslant \epsilon \left|\sigma(t) - s\right|,$$

for all $s \in U$.