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Kernel sections and uniform attractors of multi-valued semiprocesses ☆

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Abstract

We present the existence of kernel sections (which are all compact, invariant and pullback attracting) of an infinite-dimensional general multi-valued process constructed by the set-valued backward extension of multi-valued semiprocesses. Moreover, the structure of the uniform attractors of a family of multi-valued semiprocesses and the uniform forward attraction of kernel sections of a family of general multi-valued processes are investigated. Finally, we explain our abstract results by considering the mixed wave systems with supercritical exponent and ordinary differential equations.

Keywords: Multi-valued semiprocess; Kernel section; Uniform attractor; Uniform forward attraction; Nonautonomous wave system with supercritical exponent

1. Introduction

Autonomous set-valued dynamical systems and their attractors have been extensively studied in mathematical literature, especially in the recent years (see, for example, [1,3,6,7,12,14,16, 18,21,22,30] and the references cited therein). However, the nonautonomous multi-valued dynamical systems [4,23], in particular, the nonautonomous multi-valued semidynamical systems are less well understood. In this present work, we are mainly concerned with the asymptotical behavior of multi-valued semiprocesses.

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First, we give the sufficient conditions for the existence of kernel sections of an infinite-dimensional general multi-valued process which is generated by the set-valued backward extension of multi-valued semiprocesses. The existence of kernel sections of (single-valued) processes are well known. Related results can be found in [8–12,15,20,24,31,33,34], etc. As far as we know not many results in this line are available in the literature in case of nonautonomous semidynamical systems, even in single-valued case. An important reason, from the point of view of mathematics, may be that the backward extension of the semiprocesses is set-valued in the general sense. Let $\{T(h) \mid h \in \mathbb{R}^+\}$ be a continuous semigroup on a Banach space Σ with norm $\|\cdot\|_\Sigma$, and let $\{U_\sigma(t,\tau) \mid t \geqslant \tau,\ \tau \in \mathbb{R}^+\}$, $\sigma \in \Sigma$, be a family of multi-valued semiprocesses (MVSP) on a state space X satisfying the following translation identity:

$$U_{T(h)\sigma}(t,\tau) = U_\sigma(t+h,\tau+h), \quad \forall h \geqslant 0,\ t \geqslant \tau,\ \tau \geqslant 0. \tag{1.1}$$

In particular, we suppose that X is a Banach space with norm $\|\cdot\|_X$ and $U_\sigma(t,\tau)x$ is jointly norm-to-weak upper-semicontinuous in σ and x for any fixed $t \geqslant \tau$, $\tau \in \mathbb{R}^+$, i.e., if $\sigma_n \to \sigma$ and $x_n \to x$, then for any $y_n \in U_{\sigma_n}(t,\tau)x_n$, there exists a $y \in U_\sigma(t,\tau)x$, such that $y_n \rightharpoonup y$ (weak convergence), and that the semigroup $\{T(h)\}$ is continuous invariant on a subset Ξ of Σ. We will construct a general multi-valued process (GMVP) (see Section 3 for formal definitions) $\{P_\sigma(t,\tau) \mid t \geqslant \tau,\ \tau \in \mathbb{R}\}$ for each $\sigma \in \Xi$ by the following formula

$$P_\sigma(t,\tau) = \begin{cases} U_\sigma(t,\tau), & \tau \geqslant 0, \\ \widetilde{U}_\sigma(t,\tau), & \tau < 0, \end{cases} \tag{1.2}$$

where $\widetilde{U}_\sigma(t,\tau) := \{U_{\sigma'}(t-\tau,0) \mid T(|\tau|)\sigma' = \sigma\}$ in X. Let $\mathcal{K}_\sigma$ be the kernel of the GMVP $\{P_\sigma(t,\tau)\}$ with $\sigma \in \Xi$. The kernel $\mathcal{K}_\sigma$ consists of all bounded complete trajectories of the general multi-valued process, i.e.,

$$\mathcal{K}_\sigma = \Big\{u(\cdot) \mid \sup_{t\in(-\infty,+\infty)} \big\|u(t)\big\|_X \leqslant C_u,\ u(t) \in P_\sigma(t,\tau)u(\tau),\ \forall t \geqslant \tau,\ \tau \in \mathbb{R}\Big\}.$$

$\mathcal{K}_\sigma(s)$ denotes the kernel section at a time moment $s \in \mathbb{R}$:

$$\mathcal{K}_\sigma(s) = \big\{u(s) \mid u(\cdot) \in \mathcal{K}_\sigma\big\}, \quad \mathcal{K}_\sigma(s) \subseteq X.$$

Here using the technique of Kuratowski measure of noncompactness, we will show under the uniform dissipativeness and the uniform ω-limit compactness of the family of MVSPs $\{U_\sigma(t,\tau)\}$, $\sigma \in \Sigma$, that the GMVP $\{P_\sigma(t,\tau)\}$ with $\sigma \in \Xi$, which is norm-to-weak upper-semicontinuous on X, has a nonempty kernel; moreover, the kernel sections $\mathcal{K}_\sigma(t)$ are all compact, invariant ($P_\sigma(t,\tau)\mathcal{K}_\sigma(\tau) = \mathcal{K}_\sigma(t)$ for all $t \geqslant \tau$ and all $\tau \in \mathbb{R}$) and pullback attracting, i.e., for any fixed $t \in \mathbb{R}$ and every bounded set $B \subset X$,

$$\lim_{s\to+\infty} H_X^*\big(P_\sigma(t,t-s)B, \mathcal{K}_\sigma(t)\big) = 0,$$

where H_X^* denotes the Hausdorff semidistance in X. Furthermore, we give simple and sufficient methods for verifying the uniform ω-limit compactness.

Secondly, we are interested in the structure of the uniform attractors and the uniform forward attraction of the inflated kernel sections. In [12], the structure of the uniform attractors (for the