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Novel implicit/explicit local conservative schemes for the regularized long-wave equation and convergence analysis

Jiaxiang Cai [a,*], Yuezheng Gong [b], Hua Liang [a]

[a] School of Mathematical Science, Huaiyin Normal University, Huaian, Jiangsu 223300, China
[b] Beijing Computational Science Research Center, Beijing 100193, China

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ABSTRACT

Two implicit and two explicit schemes preserving the local momentum conservation laws exactly on any time–space region are proposed for the regularized long-wave equation. With appropriate boundary conditions, the schemes will be energy- and mass-preserving globally. Combining with the momentum conservation laws, we obtain the priori estimates of the numerical solution and the error estimates in l_∞ norm for the proposed implicit schemes. Numerical experiments show the excellent performance of the proposed schemes and the numerical results coincide with the theoretical ones.

1. Introduction

The regularized long-wave equation (RLW)

$$u_t + au_x - \delta u_{xxt} + (F'(u))_x = 0, \quad F(u) = \gamma u^3/6, \tag{1}$$

where a, δ and γ are positive constants, was originally introduced to describe the behavior of the undular bore by Peregrine [12]. Benjamin, Bona, and Mahony derived it as an alternative to the Korteweg–de Vries equation for describing unidirectional propagation of weakly long dispersive wave [1]. The RLW equation is very important in physics media since it describes phenomena with weak nonlinearity and dispersion waves, including nonlinear transverse waves in shallow water, ion-acoustic and magneto hydrodynamic waves in plasma and phonon packets in nonlinear crystals. The equation admits only three conservation laws given by

* Corresponding author.
E-mail addresses: cjx1981@hytc.edu.cn, thomasjeer@sohu.com (J. Cai).

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$$I_1 = \int_{-\infty}^{+\infty} u dx,\ I_2 = \int_{-\infty}^{+\infty} (u^2 + \delta u_x^2) dx,\ I_3 = \int_{-\infty}^{+\infty} \big(F(u) + au^2/2\big)\, dx, \tag{2}$$

which correspond to mass, momentum and energy respectively [11]. The RLW equation has been solved analytically for a restricted set of boundary and initial conditions. Therefore, the numerical solution of the RLW equation has been the subject of many papers [3–7,9,10,14–16,18].

In recent years, there has been an increased emphasis on constructing numerical methods to preserve certain invariant quantities in the continuous dynamical systems. The numerical method preserving at least some of structural properties of systems is called geometric integrator or structure-preserving numerical method. Nowadays, it has been a criterion to judge the success of the numerical simulation. Most of the existing energy-/momentum-preserving methods for partial differential equations (PDEs) only preserve the global energy/momentum which depends on boundary conditions. However, many PDEs admit the local conservation laws which are independent of boundary conditions, such as multi-symplectic conservation law [2]. The multi-symplectic integrators have been applied to solve the RLW equation [3,4]. They provide satisfactory numerical solution, but their performance in preservation of energy and momentum is unsatisfactory [4]. In many practical problems, the physical invariants of the system are very important, and thus we often want to use energy-/momentum-preserving schemes rather than the multi-symplectic schemes. Besides possessing the multi-symplectic conservation law, the RLW equation also admits a local energy conservation law (LECL) or a local momentum conservation law (LMCL) which is more essential than the corresponding global one [17]. By introducing some variables, the RLW equation can be reformed in a first-order form of

$$\begin{cases} \frac{1}{2}u_t + \frac{a}{2}u_x + p_x = 0, \\ \frac{1}{2}\varphi_t - \frac{\delta}{2}v_t + \frac{a}{2}\varphi_x - \frac{\delta}{2}w_x = p - F'(u), \\ u_t = w,\ u_x = v,\ \varphi_x = u, \end{cases} \tag{3}$$

where $F_t(u)/u_t$ or $F_x(u)/u_x$ can be used instead of $F'(u)$. By some simply derivations, we can prove that the RLW equation admits simultaneously a LECL

$$\big(F(u) + au^2/2\big)_t - (a\varphi_t u/2 + \varphi_t p + \delta w u_t/2)_x = 0, \tag{4}$$

and a LMCL

$$\big(u^2/2 + \delta u_x^2/2\big)_t + (up - \varphi_t u/2 - F(u))_x = 0, \tag{5}$$

which are independent of boundary conditions. Naturally, we expect to propose a scheme preserving the LECL/LMCL in the discrete sense for the RLW equation (simultaneous preservation of them is difficulty). If the idea is feasible, the proposed scheme can produce richer information on the discrete system for the RLW equation. In this paper, we mainly devote to the construction of schemes preserving the LMCL (5) rather than LECL (4). If the boundary conditions are appropriate, the proposed schemes will be momentum-/mass-preserving globally since they hold the momentum I_2 and mass I_1 in the discrete sense exactly.

The article is outlined as follows. In Section 2, two implicit and explicit schemes are proposed for the RLW equation. We prove that the schemes preserve the discrete LMCL exactly. As the RLW equation is subjected to appropriate boundary conditions, we show that the proposed schemes will be mass- and momentum-preserving globally in Section 3. The error estimates of the implicit schemes are established in Section 4. In Section 5, some numerical experiments are conducted to show the performance of the schemes and verify the theoretical analysis. Concluding remarks are given in Section 5.