J. Math. Anal. Appl. 447 (2017) 222–242

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Stability of traveling wavefronts for a discrete diffusive Lotka–Volterra competition system

Ge Tian, Guo-Bao Zhang *

College of Mathematics and Statistics, Northwest Normal University, Lanzhou, Gansu 730070, People's Republic of China

A R T I C L E I N F O

Article history:
Received 15 July 2016
Available online 12 October 2016
Submitted by J. Shi

Keywords:
Traveling wavefronts
Stability
Discrete diffusive Lotka–Volterra competition system
Weighted energy method

A B S T R A C T

In this paper, we study a discrete diffusive Lotka–Volterra competition system. It is known that this system has traveling wavefronts. We prove that the traveling wavefronts are exponentially stable, when the initial perturbation around the traveling wavefronts decays exponentially as $x \to -\infty$, but can be arbitrarily large in other locations. The approach we use here is the comparison principle and the weighted energy method.

1. Introduction

In this paper, we study the following discrete diffusive Lotka–Volterra competition system

$$\begin{cases} \frac{\partial v_1}{\partial t}(t,x) = \mathcal{D}[v_1](t,x) + r_1 v_1(t,x)[1 - v_1(t,x) - b_1 v_2(t,x)], \\ \frac{\partial v_2}{\partial t}(t,x) = \mathcal{D}[v_2](t,x) + r_2 v_2(t,x)[1 - v_2(t,x) - b_2 v_1(t,x)], \end{cases} \tag{1.1}$$

where $t > 0$, $x \in \mathbb{R}$, r_i, b_i are all positive constants, $i = 1, 2$, and

$$\mathcal{D}[v_i](t,x) = v_i(t,x+1) - 2v_i(t,x) + v_i(t,x-1).$$

This model is often used to describe the competing interaction of two species. Here $v_1(t,x)$ and $v_2(t,x)$ stand for the populations of two species at time t and location x, respectively. The parameter b_i is the competition coefficient and r_i is the net birth rate of species i, $i = 1, 2$.

* Corresponding author.
E-mail address: zhanggb2011@nwnu.edu.cn (G.-B. Zhang).

http://dx.doi.org/10.1016/j.jmaa.2016.10.012
0022-247X/

The system (1.1) is the continuum version of the following lattice dynamical system:

$$\begin{cases} \frac{\partial v_{1,j}(t)}{\partial t} = (v_{1,j+1}(t) - 2v_{1,j}(t) + v_{1,j-1}(t)) + r_1 v_{1,j}(t)(1 - v_{1,j}(t) - b_1 v_{2,j}(t)), \\ \frac{\partial v_{2,j}(t)}{\partial t} = (v_{2,j+1}(t) - 2v_{2,j}(t) + v_{2,j-1}(t)) + r_2 v_{2,j}(t)(1 - v_{2,j}(t) - b_2 v_{1,j}(t)), \end{cases} \tag{1.2}$$

where $t > 0$ and $j \in \mathbb{Z}$. Meanwhile, the system (1.1) can be regarded as a spatial discrete version of the following reaction–diffusion system:

$$\begin{cases} \frac{\partial v_1}{\partial t}(t,x) = \frac{\partial^2 v_1}{\partial x^2}(t,x) + r_1 v_1(t,x)[1 - v_1(t,x) - b_1 v_2(t,x)], \\ \frac{\partial v_1}{\partial t}(t,x) = \frac{\partial^2 v_2}{\partial x^2}(t,x) + r_2 v_2(t,x)[1 - v_2(t,x) - b_2 v_1(t,x)], \end{cases} \tag{1.3}$$

where $t > 0$ and $x \in \mathbb{R}$.

It is easy to see that the corresponding diffusionless system of (1.1)–(1.3) is

$$\begin{cases} v_1'(t) = r_1 v_1(t)[1 - v_1(t) - b_1 v_2(t)], \\ v_2'(t) = r_2 v_2(t)[1 - v_2(t) - b_2 v_1(t)]. \end{cases} \tag{1.4}$$

The system (1.4) has four constant equilibria: $(0,0)$, $(0,1)$, $(1,0)$ and coexistence equilibrium $(\frac{b_1-1}{b_1b_2-1}, \frac{b_2-1}{b_1b_2-1})$ provided that $b_1b_2 \neq 1$. By a phase plane analysis, we have the following asymptotic behaviors as $t \to +\infty$ (see [6]):

- **(i)** $(v_1, v_2) \to (1,0)$ if $0 < b_1 < 1 < b_2$.
- **(ii)** $(v_1, v_2) \to (0,1)$ if $0 < b_2 < 1 < b_1$.
- **(iii)** $(v_1, v_2) \to$ one of $(0,1)$, $(1,0)$ (depending on the initial condition) if $b_1, b_2 > 1$.
- **(iv)** $(v_1, v_2) \to (\frac{b_1-1}{b_1b_2-1}, \frac{b_2-1}{b_1b_2-1})$ (u and v coexist) if $0 < b_1, b_2 < 1$.

We need to point out that case (ii) can be reduced to the case (i) by exchanging the positions of v_1 and v_2.

The competition systems (1.2) and (1.3) have been studied quite extensively for past years, see, for example, [2–7,12,13,21] and the references cited therein. The traveling wave solution is among the central problems, since it can describe the propagation or invasion of species in population dynamics [5,6]. In mathematics, traveling wave solution of (1.2) (or (1.3)) is a special solution $(v_{1,j}(t), v_{2,j}(t)) = (\varphi_1(\xi), \varphi_2(\xi))$ with $c > 0$, $\xi := j + ct$, (or $(v_1(t,x), v_2(t,x)) = (\varphi_1(\xi), \varphi_2(\xi))$ with $c > 0$, $\xi := x + ct$), where c is the wave speed, φ_1 and φ_2 are called wave profiles. If φ_1 and φ_2 are monotone, then (φ_1, φ_2) is called a traveling wavefront. For the continuum problem (1.3), we refer the readers to the work of Hosono [9,10], Kan-on [12], Kan-on and Fang [13] and Volpert et al. [23]. For the lattice system (1.2), Guo and Wu [5] proved that there is a positive constant (the minimal wave speed) such that a traveling wavefront connecting $(0,1)$ and $(1,0)$ of (1.2) exists if and only if its speed is above this minimal wave speed. They also showed that any wave profile of (1.2) is strictly monotone and unique up to translations. It is not hard to see that (1.1) and (1.2) take the same wave profile system, i.e.,

$$\begin{cases} c\varphi_1'(\xi) = \mathcal{D}[\varphi_1](\xi) + r_1\varphi_1(\xi)[1 - \varphi_1(\xi) - b_1\varphi_2(\xi)], \\ c\varphi_2'(\xi) = \mathcal{D}[\varphi_2](\xi) + r_2\varphi_2(\xi)[1 - \varphi_2(\xi) - b_2\varphi_1(\xi)], \end{cases}$$

where $\mathcal{D}[\varphi](\xi) := \varphi(\xi+1) - 2\varphi(\xi) + \varphi(\xi-1)$. Henceforth, the existence of wavefront of (1.1) connecting $(0,1)$ and $(1,0)$ with positive speed is assured. The purpose of this article is to establish the stability of traveling wavefronts of (1.1).

The stability of traveling wavefronts for reaction–diffusion equations with monostable nonlinearity has been extensively studied, see [1,15,17–20,24–27] and reference therein. To our knowledge, there are main