J. Math. Anal. Appl. 447 (2017) 243–268

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Two-time-scales hyperbolic–parabolic equations driven by Poisson random measures: Existence, uniqueness and averaging principles

Bin Pei [a], Yong Xu [a,*], Jiang-Lun Wu [b]

[a] Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, 710072, China
[b] Department of Mathematics, Swansea University, Swansea SA2 8PP, UK

ARTICLE INFO

Article history:
Received 6 June 2016
Available online 11 October 2016
Submitted by E. Saksman

Keywords:
Averaging principles
Stochastic hyperbolic–parabolic equations
Poisson random measures
Two-time-scales

ABSTRACT

In this article, we are concerned with averaging principle for stochastic hyperbolic–parabolic equations driven by Poisson random measures with slow and fast time-scales. We first establish the existence and uniqueness of weak solutions of the stochastic hyperbolic–parabolic equations. Then, under suitable conditions, we prove that there is a limit process in which the fast varying process is averaged out and the limit process which takes the form of the stochastic wave equation is an average with respect to the stationary measure of the fast varying process. Finally, we derive the rate of strong convergence for the slow component towards the solution of the averaged equation.

1. Introduction

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual condition. Fix $L > 0$ arbitrarily, we denote $D := (0, L)$, i.e., D is a fixed, open, bounded interval of the real line $\mathbb{R}$. Let $\mathbb{H}$ denote the Hilbert space $\mathbb{L}^2(D)$ equipped with the inner product $\langle\cdot,\cdot\rangle_{\mathbb{H}}$ and the corresponding norm $\|\cdot\|$. Let $T > 0$ be fixed arbitrarily. In this paper, we are concerned with the following stochastic hyperbolic–parabolic (i.e., wave-heat) equations driven by both Brownian motions and Poisson random measures,

$$\begin{cases} \frac{\partial^2 X_t^\varepsilon(\xi)}{\partial t^2} = \Delta X_t^\varepsilon(\xi) + f(X_t^\varepsilon(\xi), Y_t^\varepsilon(\xi)) + g(X_t^\varepsilon(\xi))\dot{W}_t^1 \\ \qquad\qquad + \int_{\mathbb{Z}} h(X_{t-}^\varepsilon(\xi), z)\dot{\tilde{N}}_1(t, dz), \\ \frac{\partial Y_t^\varepsilon(\xi)}{\partial t} = \frac{1}{\varepsilon}\Delta Y_t^\varepsilon(\xi) + \frac{1}{\varepsilon}F(X_t^\varepsilon(\xi), Y_t^\varepsilon(\xi)) + \frac{1}{\sqrt{\varepsilon}}G(X_t^\varepsilon(\xi), Y_t^\varepsilon(\xi))\dot{W}_t^2 \\ \qquad\qquad + \int_{\mathbb{Z}} H(X_{t-}^\varepsilon(\xi), Y_{t-}^\varepsilon(\xi), z)\dot{\tilde{N}}_2^\varepsilon(t, dz), \\ X_t^\varepsilon(\xi) = Y_t^\varepsilon(\xi) = 0, \quad (\xi, t) \in \partial D \times (0, T], \\ X_0^\varepsilon(\xi) = X_0(\xi), \quad Y_0^\varepsilon(\xi) = Y_0(\xi), \quad \frac{\partial X_t^\varepsilon(\xi)}{\partial t}|_{t=0} = \dot{X}_0(\xi), \xi \in D, \end{cases} \tag{1.1}$$

* Corresponding author.
E-mail addresses: xgdsxxpeibin@126.com (B. Pei), hsux3@nwpu.edu.cn (Y. Xu), j.l.wu@swansea.ac.uk (J.-L. Wu).

http://dx.doi.org/10.1016/j.jmaa.2016.10.010

for $\varepsilon > 0$ and for $(\xi, t) \in D \times [0, T]$, where the drift coefficients $f(\cdot, \cdot) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $F(\cdot, \cdot) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and the diffusion coefficients $g(\cdot) : \mathbb{R} \to \mathbb{R}, G(\cdot, \cdot) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, h(\cdot, \cdot) : \mathbb{R} \times \mathbb{Z} \to \mathbb{R}, H(\cdot, \cdot, \cdot) : \mathbb{R} \times \mathbb{R} \times \mathbb{Z} \to \mathbb{R}$ are real-valued measurable functions. The detailed conditions on them will be specified in the next section. Here, $\{W_t^1\}_{t\geq 0}$ and $\{W_t^2\}_{t\geq 0}$ are given independent real-valued $\{\mathcal{F}_t\}_{t\geq 0}$-Brownian motions, and $\tilde{N}_1(dt, dz)$ and $\tilde{N}_2^\varepsilon(dt, dz)$ are compensated martingale measures associated with given mutually independent Poisson random measures $N_1(dt, dz)$ and $N_2^\varepsilon(dt, dz)$, respectively. We assume that $N_1(dt, dz)$ and $N_2^\varepsilon(dt, dz)$ are also mutually independent of $\{W_t^1\}_{t\geq 0}$ and $\{W_t^2\}_{t\geq 0}$. Before proceeding, let us explicate the Poisson random measures $\tilde{N}_1(dt, dz)$ and $\tilde{N}_2^\varepsilon(dt, dz)$. Let $(\mathbb{Z}, \mathcal{B}(\mathbb{Z}))$ be a given measurable space and $v(dz)$ be a σ-finite measure on it. Let $D_{p_t^i}, i = 1, 2$, be two countable subsets of $\mathbb{R}_+$. Furthermore, let $p_t^1, t \in D_{p_t^1}$, be a stationary $\mathcal{F}_t$-adapted Poisson point process on $\mathbb{Z}$ with characteristic v and let $p_t^2, t \in D_{p_t^2}$ be a stationary $\mathcal{F}_t$-adapted Poisson point process on $\mathbb{Z}$ with characteristic $\frac{v}{\varepsilon}$. Denote by $N^i(dt, ds)$ the Poisson random (counting) measures associated with $p_t^i, i = 1, 2$, respectively, i.e., for $i = 1, 2$

$$N^i(t, A) := \sum_{s\in D_{p_t^i}, s\leq t} I_A(p_t^i), \quad t \geq 0, \, A \in \mathcal{B}(\mathbb{Z}).$$

The corresponding compensated Poisson martingale measures are respectively defined by the following

$$\tilde{N}_1(dt, dz) := N^1(dt, dz) - v(dz)dt$$

and

$$\tilde{N}_2^\varepsilon(dt, dz) := N^2(dt, dz) - \frac{1}{\varepsilon} v(dz)dt.$$

The reader is referred to [11,25] for more detailed descriptions of the stochastic integrals with respect to (cylindrical) Wiener processes and Poisson martingale measures. It is well known nowadays that much evidence has been gathered that Poisson jumps are ubiquitous in modelling uncertainty in many diverse fields of science [1,25,12]. By now it is well established that stochastic dynamical systems driven by Poisson jump noises are much more suitable for capturing sudden bursty fluctuations, large scale moves and unpredictable events than classical diffusion modelling systems, see, e.g., [25,23,2,16].

Note that the system (1.1) is an abstract model for random vibration of an elastic string with external force on a large time scale. More generally, the slow–fast nonlinear coupled wave-heat equations could model thermoelastic wave propagations in a random medium [9], describe wave phenomena which are heat generating or are temperature related [21], as well as model biological problems with uncertainty [8,5,27]. Taking advantages of the fast and slow motions, in this paper, we focus on the limit behaviour of the slow–fast nonlinear coupled wave-heat equations driven by both Brownian motions and Poisson random measures, in which the original complex system is replaced by a much simpler averaged system.

There is an extensive literature on averaging principles for stochastic differential equations, see for example, Freidlin and Wentzell [14], Khasminskii [20], Duan [13], Thompson [26], Xu and his co-workers [28,29,32,31,33]. To obtain the effective approximation for the two-time-scales stochastic partial differential equations (SPDEs), the averaging approach for SPDEs begun to receive more attention recently. In [7], Cerrai and Freidlin showed an averaged result for stochastic parabolic equations with additive noise. In [6], Cerrai succeeded with the case of multiplicative noise. The concerned convergence in the latter two works, however, is in sense of convergence in probability (which implies weak convergence), and the rate of convergence has not been given. On the other hand, Bréhier [4] derived explicit convergence rates in both strong and weak convergences for averaging of stochastic parabolic equations. Xu, Miao and Liu [30] established averaging principles for two time-scale SPDEs driven by Poisson random measures in the sense of mean-square. Very recently, Fu et al. [18] established an averaging principle for stochastic hyperbolic–parabolic equations driven