J. Math. Anal. Appl. 447 (2017) 398–418

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Finding mixed families of special polynomials associated with Appell sequences

Subhi Khan [a,*], Nusrat Raza [b], Mahvish Ali [a,1]

[a] *Department of Mathematics, Aligarh Muslim University, Aligarh, India*
[b] *Women's College, Aligarh Muslim University, Aligarh, India*

ARTICLE INFO

Article history:
Received 31 December 2015
Available online 11 October 2016
Submitted by M.J. Schlosser

Keywords:
Laguerre–Gould Hopper based Appell polynomials
Monomiality principle
Operational techniques
Determinantal definition

ABSTRACT

In this paper, certain mixed special polynomial families associated with Appell sequences are introduced and their properties are established. Further, operational rules providing connections between these families and known special polynomials are established, which are used to derive the identities and results for the members of these new families. Determinantal definitions of the polynomials associated with Appell family are also derived. The approach presented is general.

1. Introduction and preliminaries

The concepts of the monomiality principle and operational techniques are used to combine the special polynomials to find the mixed special polynomials. These polynomials have many applications in different branches of mathematics. Recently, the Laguerre–Gould Hopper polynomials (LGHP) ${}_LH_n^{(m,r)}(x,y,z)$ are introduced in [36] which are defined by means of the generating function

$$C_0(-xt^m)\exp(yt+zt^r)=\sum_{n=0}^{\infty}{}_LH_n^{(m,r)}(x,y,z)\frac{t^n}{n!}, \tag{1.1}$$

where $C_0(x)$ denotes the Bessel–Tricomi function of order zero. The nth-order Bessel–Tricomi functions $C_n(x)$ are specified by means of the generating function

* Corresponding author.
E-mail addresses: subuhi2006@gmail.com (S. Khan), nraza.maths@gmail.com (N. Raza), mahvishali37@gmail.com (M. Ali).
[1] This work has done under Junior Research Fellowship (Award letter No. F1-17.1/2014-15/MANF-2014-15-MUS-UTT-34170/(SA-III/Website)) awarded to the third author by the University Grants Commission, Government of India, New Delhi.

http://dx.doi.org/10.1016/j.jmaa.2016.10.009

$$\exp\left(t-\frac{x}{t}\right)=\sum_{n=0}^{\infty}C_n(x)t^n, \tag{1.2}$$

for $t\neq 0$ and for all finite x and are defined by the following series [14, p. 150]:

$$C_n(x) \;=\; x^{-\frac{n}{2}}J_n(2\sqrt{x}) \;=\; \sum_{k=0}^{\infty}\frac{(-1)^k\; x^k}{k!\;(n+k)!}, \qquad n=0,1,2,\dots\;, \tag{1.3}$$

with $J_n(x)$ being the ordinary cylindrical Bessel function of the first kind [2]. The 0th-order Bessel–Tricomi function $C_0(x)$ is also given by the following operational definition:

$$C_0(\alpha x)=\exp\left(-\alpha D_x^{-1}\right)\{1\}, \tag{1.4}$$

where D_x^{-1} denotes the inverse derivative operator and

$$D_x^{-n}\{1\}=\frac{x^n}{n!}.$$

The series definition for the LGHP ${}_LH_n^{(m,r)}(x,y,z)$ is given as [36]:

$${}_LH_n^{(m,r)}(x,y,z)=n!\sum_{k=0}^{[\frac{n}{r}]}\frac{z^k\,{}_mL_{n-rk}(x,y)}{k!(n-rk)!}, \tag{1.5}$$

where ${}_mL_n(x,y)$ denotes the 2-variable generalized Laguerre polynomials (2VGLP), which are defined by the following series [18, p. 213 (27)]:

$${}_mL_n(x,y)=n!\sum_{r=0}^{[\frac{n}{m}]}\frac{x^r y^{n-mr}}{(r!)^2(n-mr)!}. \tag{1.6}$$

In view of definition (1.6), the LGHP are defined as [36, p. 9933(2.7)]:

$${}_LH_n^{(m,r)}(x,y,z)=n!\sum_{k,l=0}^{rk+ml\leq n}\frac{z^k x^l y^{n-rk-ml}}{k!(l!)^2(n-rk-ml)!}. \tag{1.7}$$

The LGHP ${}_LH_n^{(m,r)}(x,y,z)$ are also defined as [36]:

$${}_LH_n^{(m,r)}(x,y,z)=n!\sum_{k=0}^{[\frac{n}{m}]}\frac{x^k H_{n-mk}^{(r)}(y,z)}{(k!)^2(n-mk)!}, \tag{1.8}$$

where $H_n^{(r)}(y,z)$ are the Gould Hopper polynomials (GHP) [34] defined by

$$H_n^{(r)}(y,z)=n!\sum_{k=0}^{[\frac{n}{r}]}\frac{z^k y^{n-rk}}{k!(n-rk)!}. \tag{1.9}$$

The operational correspondence between the LGHP ${}_LH_n^{(m,r)}(x,y,z)$ and the generalized Laguerre Polynomials ${}_mL_n(x,y)$ is [36]: