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Holomorphically projective mappings between generalized hyperbolic Kähler spaces ☆

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ABSTRACT

We define generalized hyperbolic Kähler spaces as a particular case of Eisenhart's generalized Riemannian spaces, with some additional conditions related to the almost product structure. Since a generalized hyperbolic Kähler space is equipped with a non-symmetric basic tensor, it admits five linearly independent curvature tensors. Some properties of these curvature tensors as well as those of the corresponding Ricci tensors are established. Also, we consider holomorphically projective mappings, as well as equitorsion holomorphically projective mappings between generalized hyperbolic Kähler spaces and find some invariant geometric objects with respect to these mappings.

1. Introduction

An n-dimensional manifold M is called a *locally product space* if it admits a separating coordinate system (see [25]). This means that the manifold M is covered by a system of coordinate neighbourhoods such that in any intersection of two coordinate neighbourhoods (U, u^h) and $(U', u^{h'})$ we have

$$u^{a'} = u^{a'}(u^a), \quad u^{x'} = u^{x'}(u^x), \quad \det|\partial_a u^{a'}| \neq 0, \quad \det|\partial_x u^{x'}| \neq 0, \tag{1.1}$$

where the indices a, b, c run over the range $1, 2, \dots, p$ and the indices x, y, z run over the range $p+1, p+2, \dots, p+q = n$.

A locally product space is said to be a *hyperbolic Kähler space* if there is given a positive definite Riemannian metric and an affinor structure $F_i^h \neq \delta_i^h$ satisfying the conditions

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$$\begin{aligned} F_p^h F_i^p &= \delta_i^h, \\ g_{\alpha\beta} F_i^\alpha F_j^\beta &= -g_{ij}, \\ \nabla_k F_i^h &= 0, \end{aligned} \tag{1.2}$$

where ∇ is the operator of covariant differentiation with respect to the Levi-Civita connection of the metric g_{ij}.

As is well-known, Kähler manifolds are related to the algebra of complex numbers. In 1948, Raševskij was the first to consider a similar kind of manifolds, this time related to the algebra of double numbers and such space is called a hyperbolic Kähler space. Latterly, in 1949 Rozenfeld gave the explicit definition of para-Kähler manifolds. He compared Raševskij's definition with Kähler's definition in the complex case and remarked that Raševskij's spaces are (local) real models of para-Kähler manifolds. Much more historical remarks on para-Kähler manifolds are given in the survey paper [1].

The theory of holomorphically projective mappings between classical Kähler manifolds was started by the Japanese geometers Otsuki and Tashiro, and for a certain period of time it was one of the main research directions of the Japanese and Soviet differential geometric schools. Among Soviet geometers, some of the significant contributions to this theory have been made by Mikeš [2,5,7,8,10,9]. The theory of holomorphically projective (HP) transformations between locally product spaces was started by Prvanović [19]. As a particular case one can consider such transformations between locally decomposable Riemannian spaces and hyperbolic Kähler spaces, [21]. Among other things Prvanović [19] introduced the paraholomorphic projective curvature tensor and gave the explicit expression of the curvature tensor for spaces with constant paraholomorphic sectional curvature. Note that we have respected Prvanović's terminology by using the word "holomorphically," but strictly speaking one should use the word "paraholomorphically," to avoid any possible confusion.

Eisenhart, in his contributions to general relativity, proposed a generalization of Riemannian spaces [3,4]. This generalization consisted in using a non-symmetric basic tensor and play a fundamental role in Moffat's non-symmetric gravitational field theory [18]. Although, as is well-known, Moffat's theory is a controversial one, some modifications have improved it. Thus some hopes, based among others on results on dark matter and dark energy, have been pointed out, see for instance Janssen and Prokopec [6]. Equitorsion geodesic mappings between Eisenhart's generalized Riemannian spaces were considered in the papers [15,26,27]. So far, generalized (classical) Kähler spaces as a particular case of Eisenhart's generalized Riemannian spaces were defined and holomorphically projective mappings between such spaces were considered in the papers [16,22–24].

In the present paper we define generalized hyperbolic Kähler spaces. Also, we consider equitorsion holomorphically projective mappings between generalized hyperbolic Kähler spaces and find some invariant geometric objects with respect to these mappings. Geometric objects analogous to the Thomas projective parameter in the theory of geodesic mappings and the paraholomorphic curvature tensor in the theory of holomorphically projective mappings are examined.

2. Generalized hyperbolic Kähler spaces

On a manifold M with non-symmetric linear connection $\underset{1}{\nabla}$ another non-symmetric linear connection $\underset{2}{\nabla}$ can be defined in the following way [20]

$$\underset{2}{\nabla}_X Y = \underset{1}{\nabla}_Y X + [X, Y],\ X, Y \in T_p(M),$$

where as usual $[\cdot,\cdot]$ denotes the Lie bracket.

M. Prvanović [20] considered four curvature tensors of a non-symmetric linear connection and explained the geometric meaning of them in terms of parallel displacement with respect to the non-symmetric linear