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Permanence in nonautonomous competitive systems with nonlocal dispersal

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ABSTRACT

In this paper we consider a system with nonlocal dispersal. Applying Ahmad and Lazers definitions of lower and upper averages of a function and using the sub- and supersolution methods for PDEs we give sufficient conditions for permanence in such models. Moreover, we allow the intrinsic growth rates to be negative.

1. Introduction

In this paper we consider a nonlinear evolution system

$$\frac{\partial u_i}{\partial t} = \rho_i(\int\limits_{\Omega} K_i(x,y)u_i(t,y)dy - u_i(t,x)) + f_i(t,x,u_1 \dots, u_N)u_i, \qquad t \geq 0,\ x \in \bar{\Omega},\ i = 1,\dots,N, \tag{1.1}$$

where $u_i(t,x)$ is population density of the ith species at time t and spatial location $x \in \bar{\Omega}$, $\Omega \subset R^n$ is a compact spatial region, $\rho_i > 0$ is the dispersal rate of the ith species, $f_i(t,x,u_1,\dots,u_N)$ is the local per capita growth rate of the ith species and $K_i(\cdot,\cdot)$ is a nonlocal convolution kernel satisfying

(A1) $K_i(\cdot,\cdot) : \bar{\Omega}\times\bar{\Omega} \to R$ are positive, C^1-functions such that $\int_\Omega K_i(x,y)dy = 1$ for any $x \in \bar{\Omega}$, $i = 1,\dots,N$.

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Models of nonlocal spatial dispersal in which the dispersal operator involves an integral operator have many applications, for example, in the theory of phase transition, ecology, genetics, neurology. The kernel $K(x,y)$ may denote probability distributions (see e.g. [8,11]).

System (1.1) is the nonlocal dispersal counterpart of the following Kolmogorov competition system with random dispersal and Dirichlet boundary conditions

$$\begin{cases} \frac{\partial u_i}{\partial t} = \rho_i \Delta u_i + f_i(t,x,u_1\dots,u_N)u_i, \quad t\geq 0, x\in\Omega, \quad i=1,\dots,N, \\ u_i = 0 \quad x\in\partial\Omega. \end{cases} \tag{1.2}$$

In [6] one can find a relationship between nonlocal dispersal operators on domains with hostile surroundings and random dispersal operators on domains with Dirichlet boundary conditions.

Many important questions to ask for partial and ordinary differential equations (PDEs and ODEs for short) are permanence, persistence, extinction, global asymptotic behavior of the population. Roughly speaking, persistence means that any positive solution of the model has all its components, for large t, bounded away from zero, although, it is not excluded that the lower bound depends on the solutions. Concerning permanence, it is required that the positive lower bound is independent of the solution.

In the existing literature there are many papers which deal with sufficient conditions for permanence in ODEs (see e.g., [2,3,9,10,20]). In [21] the authors proved persistence first and next they showed global attractivity. The second notion means that for any two positive solutions their difference converges to zero as time goes to infinity. Vance and Coddington [20] considered a nonautonomus equation $u' = f(t,u)u$ with a positive initial condition. They gave sufficient conditions for this model. We use their result in the proof of the main theorem of this paper.

In [1] Ahmad and Lazer considered a Lotka–Volterra system of ODE. By introducing a notion of the upper and lower average of a function they found sufficient conditions which guarantee permanence Lotka–Volterra system of ODE. The authors proved persistence showing that a positive solution is bounded away from zero and then supposing that persistence does not hold they obtained a contradiction.

Next in [15] we extended their results to Kolmogorov system of ODE. These conditions are written as inequalities between intrinsic per capita growth rates and interaction coefficients.

The model of PDE are more realistic than model of ODE since they take into account a spatial location of species. The individuals may migrate, they may change their place where are living, etc. Because of an unfavorable conditions, the species have to find better conditions to live. In recent years models with diffusion are extensively studied (see e.g. [4,5]). One of the most popular model of PDE is reaction–diffusion parabolic partial differential equations

$$\frac{\partial u_i}{\partial t} = \mu_i \Delta u_i + f_i(t,x,u_1(t,x),\dots,u_N(t,x))u_i. \tag{1.3}$$

In [4] we found sufficient conditions for permanence in system (1.3). The conditions are expressed in terms of inequalities connecting the time averages of intrinsic growth rates, interaction coefficients, migration rates and principal eigenvalues. Moreover we showed that with the stronger inequalities the lower bound in the definition of permanence can be expressed in terms of coefficient of the system (1.3).

In this paper we investigate a system of N species Kolmogorov system with nonlocal dispersal. The models with nonlocal dispersal are considered by many researches (see e.g. [11,18,19]). In [12] authors investigated two species Lotka–Volterra competition system with nonlocal dispersal. They established a sufficient conditions for existence, uniqueness and stability of coexistence states in such systems. They also gave conditions for the extinction of one species. In models with nonlocal dispersal in contrast to the reaction–diffusion systems instead of Laplacian we have an integral operator which usually take one of the form