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Solvability of some inverse problems for the nonstationary heat-and-mass-transfer system

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A B S T R A C T

We study solvability of inverse problems of finding the right-hand side together with a solution itself for the nonstationary heat-and-mass-transfer system. The system consists of the Navier–Stokes system whose right-hand side contains the temperature of a fluid and the concentration of an admixture and the parabolic equations for the temperature of a fluid and the concentration. The right-hand side of the equation for the concentration is unknown and characterizes the volumetric density of sources of an admixture in a fluid. The usual boundary conditions are supplemented with the overdetermination conditions which are the values of the concentration on some system of surfaces.

1. Introduction

We examine the system

$$u_t - \nu\Delta u + (u,\nabla)u + \nabla p = f + \beta_c C + \beta_\theta \Theta, \quad \operatorname{div} u = 0, \tag{1}$$

$$\Theta_t - \operatorname{div}(\lambda_\theta \nabla\Theta) + (u,\nabla)\Theta = f_\theta, \tag{2}$$

$$C_t + (u,\nabla)C + (b,\nabla)C + kC - \lambda_c\,\Delta C = f_c, \tag{3}$$

where $\nu = const > 0$, $(x,t) \in Q = G \times (0,T)$ ($G \subset \mathbb{R}^n$, $T < \infty$), u, Θ, p, C are the velocity vector, the temperature of a fluid, the pressure, the concentration of an admixture (inorganic or organic) in a fluid, and f_c is the volumetric density of sources of an admixture, respectively. The system (1)–(3) describes the propagation of an admixture in a fluid. In particular, this system includes the classical Oberbeck–Boussinesq model (see, for instance, [9,22]), where the vector-function b and the function k are zero. We assume that these functions $b = (b_1, b_2, \dots, b_n)$ and k are known coefficients. Generally, this vector b (or just one coefficient before the derivative with respect to z ($z = 0$ is the surface of a fluid)) characterizes the settling

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rate for an admixture and the coefficient k the admixture decay due to chemical reactions. This more complicated model in the stationary case is studied in [2], where the relevant references can be found to both nonstationary and stationary cases. The functions f_θ and f are the densities of the heat sources and external forces. The coefficients λ_c and λ_θ stand for the solute diffusivity and the thermal diffusivity. In the Oberbeck–Boussinesq model, the vector-functions β_c and β_θ are the mass transfer coefficient and the heat-transfer coefficient multiplied by the free fall acceleration. For generality, we assume below that β_c and β_θ are vector-functions of the variables (x, t). Since in the nonlinear case we obtain only local (in time) solvability, we do not impose any constraints on n assuming just that $n \geq 2$. The proof is the same for every n. For simplicity, we assume the domain G to be bounded though the main results are valid for a wide class of unbounded domains too. The system (1)–(3) is supplemented with the initial and boundary conditions

$$u|_{t=0} = u_0, \quad u|_S = g_1(t,x), \quad \Gamma = \partial G, \; S = \Gamma \times (0,T), \tag{4}$$

$$\Theta|_{t=0} = \Theta_0, \quad \Theta|_S = g_2(t,x), \quad C|_{t=0} = C_0, \quad C|_S = g_3(t,x). \tag{5}$$

We consider an inverse problem of defining a solution to the system (1)–(3) and the right-hand side f_c in (3) using the data of additional measurements on cross-sections of G. Let $x'' = (x_{s+1}, x_{s+2}, \ldots, x_n)$ $(s = 0, 1, \ldots, n-1)$. If $s \geq 1$ then we put $x' = (x_1, x_2, \ldots, x_s)$. The right-hand side in (3) is assumed to be known in some part of the domain $Q' = G_1 \times (0,T)$ and unknown in the domain $Q'' = G_0 \times (0,T)$, where G_1 and G_0 either are nonempty disjoint domains such that $\overline{G_0} \cup \overline{G_1} = \overline{G}$ or $G_1 = \emptyset$ and thereby $Q'' = Q$. The right-hand side is of the form

$$f_c = f_0(x,t) + \sum_{i=1}^{m} f_i(x,t) a_i(x',t), \quad (x,t) \in Q, \tag{6}$$

where f_i $(i = 0, 1, \ldots, m)$ are given functions which vanish on Q' for $i = 1, 2, \ldots, m$. The functions $a_i(x', t)$ $(a_i(t)$ for $s = 0)$ in this representation are unknown and the overdetermination conditions for defining these functions are of the form

$$C|_{S_i} = \psi_i(t,x) \quad \big(S_i = (0,T) \times \Gamma_i, \quad i = 1, 2, \ldots, m\big), \tag{7}$$

where $\{\Gamma_i\}$ is a collection of smooth s-dimensional surfaces lying in G_0. For $s = 0$, the surfaces Γ_i are just points lying in G_0 and $G_0 = G$ or G_0 is a neighborhood of the union of these points. Thus we look for the unknowns $a_i(x', t)$ which depend on some part of variables and the dimension of the surfaces S_i coincides with the number $s + 1$ of these variables.

The description of some numerical methods for solving boundary value problems for the system (1)–(3) is exposed in [22]. We also can refer to the book [2], where many inverse and extremal problem are studied in the stationary case and necessary bibliography can be found. Some simplified models are studied in [20,11,7]. We do not know the articles where the inverse problems (1)–(7) for the complete system are studied. Many results connected with solvability of inverse problems for the Navier–Stokes system and the linearized Navier–Stokes system are presented in [23]. The inverse problems (1)–(7) arise when describing heat and mass transfer, filtration, diffusion, and some other physical processes. We can note that, in a real situation, even the simplest one-dimensional models used in monitoring and warning systems for river basins include several parabolic equations relative to concentrations. So our model can serve only as an example of such a model. For parabolic equations and systems the problems of the above type are studied in many articles and we can refer to the book [10], where the problems of this type are discussed in the case of parabolic equations of the second order and $s = n - 1$. In the case of $n = 1$ (thus the unknowns a_i depend only on t and the surfaces S_i are just points) linear and nonlinear problems are studied in Hölder spaces in [18].