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The frequency-localization technique and minimal decay-regularity for Euler–Maxwell equations

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A B S T R A C T

Dissipative hyperbolic systems of *regularity-loss* have been recently received increasing attention. Extra higher regularity is usually assumed to obtain the optimal decay estimates, in comparison with the global-in-time existence of solutions. In this paper, we develop a new frequency-localization time-decay property, which enables us to overcome the technical difficulty and improve the minimal decay-regularity for dissipative systems. As an application, it is shown that the optimal decay rate of $L^1(\mathbb{R}^3)$–$L^2(\mathbb{R}^3)$ is available for Euler–Maxwell equations with the critical regularity $s_c = 5/2$, that is, the extra higher regularity is not necessary.

1. Introduction

In this paper, we are interested in compressible isentropic Euler–Maxwell equations in plasmas physics (see, for example, [4,17]), which are given by the form

$$\begin{cases} \partial_t n + \nabla\cdot(nu) = 0, \\ \partial_t(nu) + \nabla\cdot(nu\otimes u) + \nabla p(n) = -n(E + u\times B) - nu, \\ \partial_t E - \nabla\times B = nu, \\ \partial_t B + \nabla\times E = 0, \end{cases} \tag{1.1}$$

with constraints

$$\nabla\cdot E = n_\infty - n, \quad \nabla\cdot B = 0 \tag{1.2}$$

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for $(t, x) \in [0, +\infty) \times \mathbb{R}^3$. Here the unknowns $n > 0, u \in \mathbb{R}^3$ are the density and the velocity of electrons, and $E \in \mathbb{R}^3, B \in \mathbb{R}^3$ denote the electric field and magnetic field, respectively. The pressure $p(n)$ is a given smooth function of n satisfying $p'(n) > 0$ for $n > 0$. For the sake of simplicity, n_∞ is assumed to be a positive constant, which stands for the density of positively charged background ions. Observe that system (1.1) admits a constant equilibrium state $(n_\infty, 0, 0, B_\infty)$, which is regarded as vector in $\mathbb{R}^{10}$. $B_\infty \in \mathbb{R}^3$ is an arbitrary fixed constant vector. The main objective of the present paper is to investigate the large-time behavior for the corresponding Cauchy problem. For this purpose, system (1.1) is supplemented with the initial data

$$(n, u, E, B)|_{t=0} = (n_0, u_0, E_0, B_0)(x), \quad x \in \mathbb{R}^3. \tag{1.3}$$

It is not difficult to see that (1.2) can hold for any $t > 0$ if the initial data satisfy the following compatible conditions

$$\nabla \cdot E_0 = n_\infty - n_0, \quad \nabla \cdot B_0 = 0, \quad x \in \mathbb{R}^3. \tag{1.4}$$

Set $w = (n, u, E, B)^\top$ ($\top$ transpose) and $w_0 = (n_0, u_0, E_0, B_0)^\top$. Then (1.1) can be written in the vector form

$$A^0(w)w_t + \sum_{j=1}^{3} A^j(w)w_{x_j} + L(w)w = 0, \tag{1.5}$$

where the coefficient matrices are given explicitly as

$$A^0(w) = \begin{pmatrix} a(n) & 0 & 0 & 0 \\ 0 & nI & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}, \; L(w) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & n(I - \Omega_B) & nI & 0 \\ 0 & -nI & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\sum_{j=1}^{3} A^j(w)\xi_j = \begin{pmatrix} a(n)(u \cdot \xi) & p'(n)\xi & 0 & 0 \\ p'(n)\xi^\top & n(u \cdot \xi)I & 0 & 0 \\ 0 & 0 & 0 & -\Omega_\xi \\ 0 & 0 & \Omega_\xi & 0 \end{pmatrix}.$$

Here, $a(n) := p'(n)/n$ is the enthalpy function, I is the identity matrix of third order. For any $\xi = (\xi_1, \xi_2, \xi_3) \in \mathbb{R}^3$, Ω_ξ is the skew-symmetric matrix defined by

$$\Omega_\xi = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix}$$

such that $\Omega_\xi E^\top = (\xi \times E)^\top$ (as a column vector in $\mathbb{R}^3$).

Clearly, (1.5) is a symmetric hyperbolic system, since $A^0(w)$ is real symmetric and positive definite and $A^j(w) (j = 1, 2, 3)$ are real symmetric. Generally, the main feature of (1.5) is the finite time blowup of classical solutions even when the initial data are smooth and small. In one dimensional space, Chen, Jerome and Wang [4] first constructed global weak solutions by using the Godunov scheme of the fractional step. By using the dissipative effect of damping terms, Peng Wang and Gu [28] established the global existence of smooth solutions in the periodic domain. Duan [7] analyzed the regularity-loss mechanism in the dissipation