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Uniform stabilization of the fourth order Schrödinger equation

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A B S T R A C T

We study both boundary and internal stabilization problems for the fourth order Schrödinger equation in a smooth bounded domain Ω of $\mathbb{R}^n$. We first consider the boundary stabilization problem. By introducing suitable dissipative boundary conditions, we prove that the solution decays exponentially in an appropriate energy space. In the internal stabilization problem, by assuming that the damping term is effective on a neighborhood of a part of the boundary, we prove the exponential decay of the $L^2(\Omega)$-energy of the solution. Both results are established by using multiplier techniques and compactness/uniqueness arguments.

1. Introduction

Let Ω be an open bounded domain of $\mathbb{R}^n$ with sufficiently smooth boundary Γ. Let $\{\Gamma_0, \Gamma_1\}$ be a partition of Γ defined by

$$\Gamma_0 = \{x \in \Gamma, m(x) \cdot \nu(x) > 0\}, \tag{1}$$

$$\Gamma_1 = \{x \in \Gamma, m(x) \cdot \nu(x) \le 0\} \tag{2}$$

where $\nu(.)$ is the unit normal vector to Γ pointing towards the exterior of Ω, $m(x) = x - x_0$, and x_0 is a fixed point in the exterior of Ω such that

$$\overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \varnothing. \tag{3}$$

In Ω, we consider the fourth order Schrödinger equation with boundary damping term supported on Γ_0

$$\frac{\partial y(x,t)}{\partial t} = i\Delta^2 y(x,t) \qquad \text{in } \Omega \times (0, +\infty), \tag{4}$$

$$y(x,0) = y_0(x) \qquad \text{in } \Omega, \tag{5}$$

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$$y(x,t) = \frac{\partial y(x,t)}{\partial \nu} = 0 \qquad \text{on } \Gamma_1 \times (0,+\infty), \tag{6}$$

$$\Delta y(x,t) = 0 \qquad \text{on } \Gamma_0 \times (0,+\infty), \tag{7}$$

$$\frac{\partial \Delta y(x,t)}{\partial \nu} = m(x)\cdot\nu(x)\frac{\partial y(x,t)}{\partial t} \qquad \text{on } \Gamma_0 \times (0,+\infty). \tag{8}$$

The natural energy space for system (4)–(8) is the space

$$V = \{f \in H^2(\Omega); f = \frac{\partial f}{\partial \nu} = 0 \text{ on } \Gamma_1\}$$

endowed with the norm induced by the inner product

$$< f, g > = \int_\Omega \Delta f(x)\Delta\overline{g(x)}dx$$

which in V is equivalent to the H^2-norm. Thus the energy function of a solution of system (4)–(8) is

$$E(t) = \frac{1}{2}\|y(t)\|_V^2 = \frac{1}{2}\int_\Omega |\Delta y(x,t)|^2\, dx.$$

Regarding the well-posedness of the solutions to the system (4)–(8), we have the following result.

Theorem 1. *For any initial datum $y_0 \in V$, system (4)–(8) has a unique solution*

$$y \in C([0,+\infty); V) \cap C^1([0,+\infty), V').$$

Here V' is the dual of V. Moreover if $y_0 \in H^6(\Omega) \cap V$, and

$$\Delta y_0(x) = 0 \quad on\ \Gamma_0,$$
$$\frac{\partial \Delta y_0(x)}{\partial \nu} = im(x)\cdot\nu(x)\Delta^2 y_0(x) \quad on\ \Gamma_0,$$

then $y \in C^1([0,+\infty); V) \cap C([0,+\infty); H^5(\Omega) \cap V)$ and satisfies

$$\Delta y(x,t) = 0 \quad on\ \Gamma_0 \times (0,+\infty),$$
$$\frac{\partial \Delta y(x,t)}{\partial \nu} = im(x)\cdot\nu(x)\Delta^2 y(x,t) \quad on\ \Gamma_0 \times (0,+\infty).$$

In the following theorem we state an exponential stability result for system (4)–(8).

Theorem 2. *There exist positive constants M and δ such that for any initial datum $y_0 \in V$, the energy $E(.)$ of the solution of the system (4)–(8) where Γ_0 and Γ_1 are given by (1) and (2) satisfies the inequality*

$$E(t) \le Me^{-\delta t}E(0) \tag{9}$$

for all $t \ge 0$.