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# The lack of polynomial stability to mixtures with frictional dissipation

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A B S T R A C T

We consider the system modeling a mixture of $n$ materials with frictional damping. We show that the corresponding semigroup is exponentially stable if and only if the imaginary axis is contained in the resolvent set of the infinitesimal generator. In particular this implies the lack of polynomial stability to the corresponding semigroup.



## 1. Introduction

The theory of mixtures of solids has been widely investigated in the last decades, see for example [5,6,8,9,11–13,24,26]. In recent years, an increasing interest has been directed to the study of the qualitative properties of solutions related to mixtures composed of two interacting continua. Several results concerning existence, uniqueness, continuous dependence and asymptotic stability can be found in the literature [1–4,11,15,17–19,22,23]. In [10] F. Dell'Oro and Rivera, made a full characterization of the asymptotic behavior of the following mixture model

$$\rho_1 u^1_{tt} = a_{11}u^1_{xx} + a_{12}u^2_{xx} + a_{13}u^3_{xx} - \alpha(u^1 - u^2 + u^3) - d_1 u^1_t,$$
$$\rho_2 u^2_{tt} = a_{12}u^1_{xx} + a_{22}u^2_{xx} + a_{23}u^3_{xx} + \alpha(u^1 - u^2 + u^3) - d_2 u^2_t,$$

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$$\rho_3 u^3_{tt} = a_{13}u^1_{xx} + a_{23}u^2_{xx} + a_{33}u^3_{xx} - \alpha(u^1 - u^2 + u^3) - d_3 u^3_t,$$

with $d_i \geq 0$, where at least one $d_i$ is positive. They proved that depending on the relationships of the coefficients, three situations may occur. The system can be exponentially stable, polynomially stable or there exists oscillating solution.

Here we study the one dimensional model of a mixture of $n$ interacting continua with reference configuration over $[0, l]$. Let us denote by $u^1 := u^1(x_1, t),\ u^2 := u^2(x_2, t),\ \dots, u^n := u^n(x_n, t)$ where $x_i \in [0, l]$. We assume that the particles under consideration occupy the same position at time $t = 0$, so that $x = x_i$, therefore we can assume that

$$u^i : \ [0, l] \times [0, \infty) \to \mathbb{R}.$$

The corresponding motion equations are given by

$$\rho_i u^i_{tt} = T^i_x + P^i + F^i, \quad i = 1, \cdots n, \tag{1}$$

where $\rho_i$ denotes the mass density, $T^i$ is the stress contribution of the $i$ component of the mixture, $P^i$ is the internal body force that depends on the relative displacements $(u^1, \cdots, u^n)$ and $F^i$ stand for the external forces associated with the constituents $(u^i)$. The constitutive law we use is

$$T^i = a_{i1}u^1_x + a_{i2}u^2_x + \dots + a_{in}u^n_x, \quad i = 1, \cdots n. \tag{2}$$

Here we assume that $P^i$ is small such that it can be neglected and the frictional dissipative mechanism is produced as the external source given by

$$F^i = -b_{i1}u^1_t - b_{i2}u^2_t - \dots - b_{in}u^n_t, \quad i = 1, \cdots n. \tag{3}$$

Substituting relations (2)–(3) into system (1) we get

$$\mathbf{R}U_{tt} - \mathbf{A}U_{xx} + \mathbf{B}U_t = 0, \tag{4}$$

with $U = (u^1, \cdots, u^n)$ and

$$\mathbf{R} = (\rho_i \delta_{ij})_{n\times n}, \quad \mathbf{A} = (a_{ij})_{n\times n}, \quad \mathbf{B} = (b_{ij})_{n\times n}$$

where $\delta_{ij}$ is the Kronecker's delta, $\mathbf{A}$ is a positive definite (real) symmetric matrix and $\mathbf{B}$ a semipositive definite (real) symmetric matrix. The initial conditions are given by

$$U(x, 0) = U_0(x), \quad U_t(x, 0) = U_1(x). \tag{5}$$

Finally, we consider Dirichlet boundary conditions

$$U(0, t) = U(l, t) = 0, \quad t \in \mathbb{R}^+. \tag{6}$$

In that follows and without loss of generality we can assume that $\mathbf{B}$ is diagonal matrix. Otherwise we make the substitution $U = \mathbf{S}^T\widetilde{U}$ in equation (4), where $\mathbf{S}^T$, the transpose $\mathbf{S}$, is the nonsingular matrix that diagonalize $\mathbf{R}$ and $\mathbf{B}$ simultaneously (see the Theorem 1). Multiplying the resulting equation by the matrix $\mathbf{S}$ we have

$$\mathbf{SRS}^T\widetilde{U}_{tt} - \mathbf{SAS}^T\widetilde{U}_{xx} + \mathbf{SBS}^T\widetilde{U}_t = 0,$$