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Finite simple labeled graph C^*-algebras of Cantor minimal subshifts

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It is well known that a simple graph C^*-algebra is either AF or purely infinite. In this paper, we address the question of whether this is the case for labeled graph C^*-algebras which include all graph C^*-algebras and Matsumoto algebras of subshifts. There have been various C^*-algebra constructions associated with subshifts and some of them are known to have the crossed products $C(X) \times_T \mathbb{Z}$ of Cantor minimal subshifts (X, T) as their quotient algebras.
We show that such a simple crossed product $C(X) \times_T \mathbb{Z}$ can be realized as a labeled graph C^*-algebra. Since this C^*-algebra is known to be an $A\mathbb{T}$ algebra and has $\mathbb{Z}$ as its K_1-group, our result provides a family of simple finite non-AF unital labeled graph C^*-algebras.

1. Introduction

There has been significant interaction between operator algebras and dynamical systems since the Cuntz–Krieger algebras $\mathcal{O}_A$ [12] were introduced as invariants of subshifts of finite type with transition matrices A. The idea of associating a C^*-algebras to a subshift of finite type and the method to calculate its K-groups have been extensively studied in more general context of subshifts by various authors (see [8,10,28] among many others). Several classes of C^*-algebras were constructed from one-sided or two-sided subshifts and their structure theory was investigated producing many interesting results. For example, the C^*-algebra obtained in [10] from a one-sided subshift X^+ corresponding to a two-sided subshift X has as its quotient C^*-algebra the crossed product $C(X) \times_T \mathbb{Z}$ by the shift transform T on X. On the other hand, the Cuntz–

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Krieger algebra $\mathcal{O}_A$ can also be viewed as a universal C^*-algebra generated by partial isometries satisfying the relations imposed from the matrix A. Since A is a vertex (or edge) matrix of a finite directed graph, this way of characterizing the Cuntz–Krieger algebra has been naturally generalized to the C^*-algebras of inifinite (ultra) graphs, infinite matrices, higher-rank graphs, and labeled graphs. One purpose of the present paper is to show that the crossed product $C(X) \times_T \mathbb{Z}$ of a two-sided subshift (X, T) can be realized as a labeled graph C^*-algebra whenever the crossed product is simple (or equivalently whenever the subshift (X, T) is minimal).

With the motivation to provide a common framework for studying the ultragraph C^*-algebras [31,32] and the shift space C^*-algebras (see [7,9,28] among others), Bates and Pask [3] introduced the C^*-algebras associated to labeled graphs (more precisely, labeled spaces). Graph C^*-algebras (see [2,5,26,27,30] among many others) and Exel–Laca algebras [15] are ultragraph C^*-algebras and all these algebras are defined as universal objects generated by partial isometries and projections satisfying certain relations determined by graphs (for graph C^*-algebras), ultragraphs (for ultragraph C^*-algebras), and infinite matrices (for Exel–Laca algebras). In a similar but more complicated manner, a labeled graph C^*-algebra $C^*(E, \mathcal{L}, \mathcal{B})$ is also defined as a C^*-algebra generated by partial isometries $\{s_a : a \in \mathcal{A}\}$ and projections $\{p_A : A \in \mathcal{B}\}$, where $\mathcal{A}$ is an alphabet onto which a *labeling map* $\mathcal{L} : E^1 \to \mathcal{A}$ is given from the edge set E^1 of the directed graph E, and $\mathcal{B}$, an *accommodating set*, is a set of vertex subsets $A \subset E^0$ satisfying certain conditions. The family of these generators is assumed to obey a set of rules regulated by the triple $(E, \mathcal{L}, \mathcal{B})$ called a *labeled space* and moreover it should be universal in the sense that any C^*-algebra generated by a family of partial isometries and projections satisfying the same rules must be a quotient algebra of $C^*(E, \mathcal{L}, \mathcal{B})$. The universal property allows the group $\mathbb{T}$ to act on $C^*(E, \mathcal{L}, \mathcal{B})$ in a canonical way, and this action γ (called the *gauge action*) plays an important role throughout the study of generalizations of the Cuntz–Krieger algebras. The Cuntz–Krieger algebras [12] (and the Cuntz algebras [11]) are the C^*-algebras of finite graphs from which, as mentioned above, many generalizations have emerged in various ways including the C^*-algebras of higher-rank graphs whose study started in [25].

Simplicity and pure infiniteness results for labeled graph C^*-algebras are obtained in [4], and in particular it is shown that there exists a purely infinite simple labeled graph C^*-algebra which is not isomorphic to any graph C^*-algebras. Thus we can say that the class of simple labeled graph C^*-algebras is strictly larger than that of simple graph C^*-algebras. As is shown in [32], every simple ultragraph C^*-algebra is either AF or purely infinite, whereas we know from [29] that among higher rank graph C^*-algebras there exist simple C^*-algebras which are neither AF nor purely infinite. More specifically there exist such simple C^*-algebras which are stably isomorphic to irrational rotation algebras or Bunce–Deddens algebras. These examples of finite (but non-AF) simple C^*-algebras associated to higher rank graphs raise a natural question of whether there exist labeled graph C^*-algebras that are simple finite but non-AF. In this paper we answer this question positively by providing a family of such simple labeled graph C^*-algebras that are isomorphic to crossed products $C(X) \times_T \mathbb{Z}$ of Cantor minimal systems (X, T), where the compact metric spaces X are subshifts over finite alphabets.

A dynamical system (X, T) consists of a compact metrizable space X and a transformation $T : X \to X$ which is a homeomorphism. This determines a C^*-dynamical system $(C(X), \mathbb{Z}, T)$ where $T(f) := f \circ T^{-1}$, $f \in C(X)$ and thus gives rise to the crossed product $C(X) \times_T \mathbb{Z}$. If two dynamical systems (X_i, T_i), $i = 1, 2$, are topologically conjugate, namely if there is a homeomorphism $\phi : X_1 \to X_2$ satisfying $T_2(\phi(x)) = \phi(T_1(x))$ for all $x \in X$, then it is rather obvious that the crossed products are isomorphic. As a consequence of the Markov–Kakutani fixed point theorem, one can show that there exists a Borel probability measure m on X which is T-invariant in the sense that $m \circ T^{-1} = m$ (for example, see [13, Theorem VIII. 3.1]). If there exists a unique T-invariant measure, we call (X, T) *uniquely ergodic*. If X is the only non-empty closed T-invariant subspace of X, the system (X, T) is said to be *minimal*, and as is well known, a dynamical system (X, T) is minimal if and only if each T-orbit $\{T^i x : i \in \mathbb{Z}\}$, $x \in X$, is dense in X. A Cantor space is characterized as a compact metrizable totally disconnected space with no isolated points, and a dynamical system (X, T)