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Ergodicity of stochastic Magneto-Hydrodynamic equations driven by α-stable noise ☆

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A B S T R A C T

The current paper is devoted to the ergodicity of stochastic Magneto-Hydrodynamic equations driven by α-stable noise with $\alpha \in (\frac{3}{2}, 2)$. By the maximal inequality for the stochastic α-stable convolution and vorticity transformation, the well-posedness of the mild solution for stochastic Magneto-Hydrodynamic equation is established. Due to the discontinuous trajectories, the existence and uniqueness of the invariant measure for stochastic Magneto-Hydrodynamic equation are obtained by the strong Feller property and the accessibility to zero instead of the irreducibility.

1. Introduction

In recent years, stochastic partial differential equations (SPDEs) driven by Lévy noise have attracted a lot of attention, see [1,7,19,22,26] and references therein. But in these works, Lévy noises are assumed to be square integrable which clearly rules out the interesting α-stable noises. This restriction should be relaxed since α-stable noises have been deeply studied and widely applied to physics, queueing theory, mathematical finances and others. There are a few papers on stochastic partial differential equation driven by α-stable noises (see for instance [9,22,21,20,29–31]). The authors in [22] investigated the structural properties of solutions to the nonlinear stochastic equations with bounded and Lipschitz nonlinearities driven by cylindrical stable processes. While [25] studied the ergodicity of the stochastic equation with unbounded and non-Lipschitz dissipative function driven by α-stable noises with $\alpha \in (1, 2)$. The exponential mixing of the SPDEs driven by α-stable noises has been established in [21,20,31]. Dong, Xu & Zhang in [9] proved the exponential ergodicity and strong Feller of the stochastic Burgers equations driven by $\frac{\alpha}{2}$-subordinated cylindrical Brownian motions with $\alpha \in (1, 2)$. The existence of the invariant measure has been shown for 2D stochastic Navier–Stokes equation forced by α-stable noises with $\alpha \in (1, 2)$, see [7] for details. Recently,

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Xu in [29] studied the ergodicity of the stochastic real Ginzburg–Landau equation driven by α-stable noises with $\alpha \in (\frac{3}{2}, 2)$ and established a maximal inequality for the stochastic α-stable convolution which is useful for studying other SPDEs forced by α-stable noises.

The dynamics of the velocity and the magnetic field in electrically conducting fluids and some basic physics conservation laws can be described by the magneto-hydrodynamic (MHD) equations. More details of the related background can be found in [4,6,16]. There has been extensive study of the following MHD equations

$$\begin{cases} \partial_t u + u \cdot \nabla u = -\nabla p + \nu_1 \partial^2_{x_1} u + \nu_2 \partial^2_{x_2} u + b \cdot \nabla b, \\ \partial_t b + u \cdot \nabla b = \eta_1 \partial^2_{x_1} b + \eta_2 \partial^2_{x_2} b + b \cdot \nabla u, \\ \nabla \cdot u = 0, \quad \nabla \cdot b = 0, \\ u(0,x) = u_0, \quad b(0,x) = b_0, \end{cases} \tag{1.1}$$

where $(x_1, x_2) \in \mathbb{R}^2$, $t \geq 0$, $u = (u_1, u_2)$ and $b = (b_1, b_2)$ denote the velocity field and magnetic field respectively, p is a scalar pressure, $\nu_1, \nu_2 \geq 0$ is the kinematic viscosity, $\eta_1, \eta_2 \geq 0$ is the magnetic diffusion. Sermange and Temam in [23], and Duvant and Lions in [10] showed the existence and uniqueness of the global solution corresponding to the sufficiently smooth initial data for (1.1) for all parameters $\nu_1, \nu_2, \eta_1, \eta_2 > 0$, see Theorem 6 in [10]. Also, when some of the parameters are positive, the well-posedness of (1.1) was obtained in [5,17,35]. For more details of the regularity for MHD systems, we refer the reader to [11,15,28,34,35,33,36]. For the two dimensional stochastic MHD equations

$$\begin{cases} dX = (\nu \Delta X - (X \cdot \nabla)X + S(B \cdot \nabla)B - \nabla(P + \frac{1}{2}S|B|^2)dt + \sqrt{Q_1}dW_1(t), \\ dB = (\nu_1 \Delta B - (X \cdot \nabla)B + (B \cdot \nabla)X)dt + \sqrt{Q_2}dW_2(t), \\ \nabla \cdot X = 0, \quad \nabla \cdot B = 0, B \cdot n = 0, \quad \text{in} \quad (0, +\infty) \times \mathbb{O}, \\ X = 0, \quad curl B = 0, \quad \text{on} \quad (0, +\infty) \times \partial\mathbb{O}, \\ X(0,\xi) = x_0(\xi), \quad B(0,\xi) = b_0(\xi), \quad \text{on} \quad \mathbb{O}, \end{cases} \tag{1.2}$$

Barbu and Da Prato in [3] showed the existence of the solution to the stochastic MHD equations (1.2), and proved the existence and uniqueness of an invariant measure by the coupling methods. Recently, Huang and Shen in [14] studied the well-posedness and dynamics of the stochastic 2D incompressible fractional MHD equation driven by Gaussian white noise. Manna and Mohan in [18] studied the incompressible, viscous and non-resistive MHD equations with Levy noise, and proved local in time existence and pathwise uniqueness of strong solution, and obtained the invariant measures. Very recently, Wang et al. in [27] established the existence of the martingale solution for the stochastic compressible Navier–Stokes equation with Brownian motions, Sun and Li in [24] established the existence of weak solutions for stochastic compressible MHD equations. There are other relative works on stochastic compressible fluid flows, see for instance [12,32].

Motivated by the work in [3,14,29], we consider the following stochastic 2D MHD equation in tours $\mathbb{T}^2 = (0,1]^2$

$$\begin{cases} du = [\Delta u + b \cdot \nabla b - u \cdot \nabla u - \nabla p]dt + dL_t, \quad t \geq 0, \\ db = [\Delta b + b \cdot \nabla u - u \cdot \nabla b]dt + dL_t, \quad t \geq 0, \\ \nabla \cdot u = 0, \quad \nabla \cdot b = 0, \\ u(0,x) = u_0, \quad b(0,x) = b_0, \end{cases} \tag{1.3}$$

where L_t is cylindrical α-stable noise (be specialized later).

In this paper, we consider the ergodicity of equation (1.3) with $\alpha \in (\frac{3}{2}, 2)$. The maximal inequality for the stochastic α-stable convolution is applied, which is developed by Xu in [29], vorticity transformation and