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Almost periodic evolution systems with impulse action at state-dependent moments

Robert Hakl [a,d], Manuel Pinto [b], Viktor Tkachenko [c], Sergei Trofimchuk [d,*]

[a] *Institute of Mathematics, AS CR, Prague, Czech Republic*
[b] *Facultad de Ciencias, Universidad de Chile, Santiago, Chile*
[c] *Institute of Mathematics, National Academy of Sciences of Ukraine, Tereshchenkivs'ka str. 3, Kyiv, Ukraine*
[d] *Instituto de Matemática y Fisica, Universidad de Talca, Casilla 747, Talca, Chile*

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ABSTRACT

We study the existence of almost periodic solutions for semi-linear abstract parabolic evolution equations with impulse action at state-dependent moments. In particular, we present conditions excluding the beating phenomenon in these systems. The main result is illustrated with an example of impulsive diffusive logistic equation.

1. Introduction

The studies of almost periodic and almost automorphic solutions constitute a significant part of the theory of impulsive systems. Already in their seminal work of 1968, "Teoria calitativa a sistemelor cu impulsuri", A. Halanay and D. Wexler elaborated a framework showed to be adequate to approach the topic of almost periodicity in different contexts of the theory of discontinuous semi-dynamical systems. One of cornerstones of this framework was the concept of an almost periodic measure [25], in the posterior works usually reduced to a simpler subclass of Wexler's almost periodic measures, cf. [32]. The Halanay and Wexler's book [7] triggered the interest of various researchers in the field of differential equations, and the monographs [15, 21,27,32] present the main achievements of the almost periodic impulsive theory obtained during the last decades of the past century. As it was shown in [24,32,33], in the case when the consecutive moments t_j of impulse action are uniformly separated, i.e. $\inf_j\{t_{j+1} - t_j\} \geq \theta > 0$, almost periodic impulsive equations generate equivalent continuous semi-dynamical systems. In consequence, various principles (e.g. formulated

* Corresponding author.
E-mail addresses: hakl@ipm.cz (R. Hakl), pintoj@uchile.cl (M. Pinto), vitk@imath.kiev.ua (V. Tkachenko), trofimch@inst-mat.utalca.cl (S. Trofimchuk).

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by Favard, Levitan, Zhikov) of the classical almost periodic theory are also valid for the impulsive case. In the latter context, sometimes it is convenient to replace Wexler's concept of a piecewise continuous almost periodic function with more simple definition of Stepanov almost periodic solution [24,32,33]. The case when $\inf_j\{t_{j+1}-t_j\}=0$ is much more complicated and can produce various surprising effects. For instance, an exponentially dichotomic almost periodic linear inhomogeneous system typically does not possess any almost periodic solution. Nevertheless, it still has a unique essentially automorphic (more precisely, Levitan N-almost periodic) bounded solution [24].

The recent years have again witnessed a growth of interest in the theory of almost periodic impulsive systems (and their applications as well, cf. [6,17,27,35]). First, because of an interesting and promising connection between almost periodic dynamic equations on time scales and almost periodic impulsive systems [14]. It seems that the studies of general almost periodic time scales [34] can benefit from the general theory of almost periodic sets on the real line [23,32]. Second, we would like to mention a series of recent studies of topological impulsive semiflows, where the concept of an almost periodic motion plays one of the central roles. See [3–5] for more details and references. And, finally, the subclass of abstract impulsive systems seems now to be attracting much more interest from the experts in the field, e.g. see [1,3,8,10,11,16,26, 28,29,31]. Periodic and almost periodic solutions of these systems were investigated by many authors, we direct the reader to [8,15,18,20,28,31,33] for some relevant results and further references. Due to the complex nature of abstract almost periodic impulsive equations, they always were considered with pre-fixed (i.e. state-independent) moments of impulse action. However, these moments may depend on the current state of the evolutionary process [2,19–21] that requires the analysis of almost periodic evolution systems with impulse action at variable times. In the present work, we are doing the first step in this direction, by investigating the abstract almost periodic system

$$\frac{dx}{dt}+(A+A_1(t))x=f(t,x),\quad t\neq\tau_j(x(t)),\tag{1}$$

$$x(t+0)-x(t)=g_j(x(t)),\quad t=\tau_j(x(t)),\ j\in\mathbb{Z},\tag{2}$$

having impulsive forces located on the surfaces $\Gamma_j:=\{(t,x):t=\tau_j(x)\}$ which are uniformly separated each from other. Here $x(t),\ t\in\mathbb{R}$, belongs to a Banach space X, A is a sectorial operator in X, and closed operators $A_1(t),\ t\in\mathbb{R}$, are generally unbounded in X. Our main goal is to develop a new approach to the main challenges appearing in the studies of system (1), (2): (a) the fluctuation of points of discontinuities from one solution of (1), (2) to another; (b) the beating phenomenon, when a trajectory of (1), (2) may hit the same surface Γ_j several times; (c) an adequate election of functional spaces, in order to obtain 'sufficiently strong' almost periodic solutions. It should be observed here that in many works the possibility of beatings of trajectories is usually excluded at the very beginning of studies. It can be reached by assuming rather strong restrictions on the pairs $\{\tau_j,g_j\}$, cf. [4]. To simplify our exposition, we will also exclude the beatings; nevertheless, our restrictions seem to be rather moderate from the geometrical point of view, see Lemma 3 below. In any case, it is completely natural (at least, from the perspective of applications in mechanics) to ask about the existence of almost periodic regimes with beatings [32]. Following the tradition, we will also invoke the usual definition of Wexler's piecewise continuous almost periodic function. This requires auxiliary results similar to Lemma 1 (a 'prototype' version of which can be found in [7]: here we include the proof of this lemma for the completeness). The main result of this paper is Theorem 8 in the third section. It states the existence of an almost periodic solution to system (1), (2) under the assumptions of sufficient roughness of its linear part and smallness of the Lipschitz constants for all nonlinearities $f(t,x),\tau_j(x),g_j(x)$ in the variable x. The aforementioned roughness is expressed in terms of the exponential dichotomies of evolution systems [9,30], and some properties of the associated Green functions are analysed in Section 2.3.

To prove the existence of almost periodic solution, we follow the strategy proposed in [19] for the abstract periodic impulsive systems (1), (2) (and which differs from the reduction method proposed in [2] for the