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An analysis of a mathematical model describing the geographic spread of dengue disease

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ABSTRACT

We consider a system of nonlinear partial differential equations corresponding to a generalization of a mathematical model for geographical spreading of dengue disease proposed in the article by Maidana and Yang (2008) [5]. As in that article, the mosquito population is divided into subpopulations: winged form (mature female mosquitoes) and aquatic form (comprising eggs, larvae and pupae); the human population is divided into the subpopulations: susceptible, infected and removed (or immune). On the other hand, differently from the work by Maidana and Yang, who considered just the one dimensional case with constant coefficients, in the present we allow higher spatial dimensions and also parameters depending on space and time. This last generalization is done to cope with possible abiotic effects as variations in temperature, humidity, wind velocity, carrier capacities, and so on; thus, the results hold for more realistic situations. Moreover, we also consider the effects of additional control terms. We perform a rigorous mathematical analysis and present a result on existence and uniqueness of solutions of the problem; furthermore, we obtain estimates of the solution in terms of certain norms of the given parameters of the problem. This kind of result is important for the analysis of optimal control problems with the given dynamics; to exemplify their utility, we also briefly describe how they can be used to show the existence of optimal controls that minimize a given optimality criteria.

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1. Introduction

The main objective of this work is to perform a rigorous mathematical analysis of a system of nonlinear partial differential equations corresponding to a generalization of a mathematical model for geographical spreading of dengue disease proposed by Maidana and Yang in [5].

To describe the model, let $\Omega \subset \mathbb{R}^n$, $n = 1, 2, 3$, be an open and bounded set associated to a geographical region where a certain human population is settled and the *Aedes aegypti* mosquito population is spreading; let also $0 < \bar{T} < \infty$ be a given final time of interest and denote by $\bar{t}$ the times between $[0, \bar{T}]$ and $\bar{Q} = \Omega \times (0, \bar{T})$, the space–time cylinder and $\bar{\Gamma} = \partial\Omega \times (0, \bar{T})$, the space–time boundary. Then, the system of equations we are considering is the following:

$$\begin{cases} \dfrac{\partial \bar{M}_S}{\partial \bar{t}} = \displaystyle\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(\bar{D}_{ij}^M \frac{\partial \bar{M}_S}{\partial x_j}\right) - \sum_{i=1}^{n} \bar{v}_i \frac{\partial \bar{M}_S}{\partial x_i} \\ \qquad\qquad + \bar{\gamma}\bar{A}\left(1 - \dfrac{\bar{M}}{\bar{k}_1}\right) - \bar{\mu}_1 \bar{M}_S - \bar{\beta}_1 \bar{M}_S \bar{I} - \bar{h}_1 \bar{M}_S \mathbf{1}_{\omega_1}, \\ \dfrac{\partial \bar{M}_I}{\partial \bar{t}} = \displaystyle\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(\bar{D}_{ij}^M \frac{\partial \bar{M}_I}{\partial x_j}\right) - \sum_{i=1}^{n} \bar{v}_i \frac{\partial \bar{M}_I}{\partial x_i} - \bar{\mu}_1 \bar{M}_I \\ \qquad\qquad + \beta_1 \bar{M}_S \bar{I} - \bar{h}_1 \bar{M}_I \mathbf{1}_{\omega_1}, \\ \dfrac{\partial \bar{A}}{\partial \bar{t}} = \bar{r}\left(1 - \dfrac{\bar{A}}{\bar{k}_2}\right)\bar{M} - \bar{\mu}_2 \bar{A} - \bar{\gamma}\bar{A} - \bar{h}_2 \bar{A}\mathbf{1}_{\omega_2}, \\ \dfrac{\partial \bar{H}}{\partial \bar{t}} = \bar{\mu}_H \bar{N} - \bar{\mu}_H \bar{H} - \bar{\beta}_2 \bar{H}\bar{M}_I, \\ \dfrac{\partial \bar{I}}{\partial \bar{t}} = \bar{\beta}_2 \bar{H}\bar{M}_I - \bar{\sigma}\bar{I} - \bar{\mu}_H \bar{I}, \\ \dfrac{\partial \bar{R}}{\partial \bar{t}} = \bar{\sigma}\bar{I} - \bar{\mu}_H \bar{R}, \end{cases} \tag{1.1}$$

holding in $\bar{Q}$, together with the following given boundary and initial conditions:

$$\begin{cases} \dfrac{\partial \bar{M}_S}{\partial \eta_{\bar{D}^M}}(\cdot) = \dfrac{\partial \bar{M}_I}{\partial \eta_{\bar{D}^M}}(\cdot) = 0 & \text{on} \quad \bar{\Gamma}, \\ \bar{M}_S(\cdot, 0) = \bar{M}_{S0}(\cdot) & \text{on} \quad \Omega \\ \bar{M}_I(\cdot, 0) = \bar{M}_{I0}(\cdot) & \text{on} \quad \Omega \\ \bar{A}(\cdot, 0) = \bar{A}_0(\cdot) & \text{on} \quad \Omega, \\ \bar{H}(\cdot, 0) = \bar{H}_0(\cdot) & \text{on} \quad \Omega, \\ \bar{I}(\cdot, 0) = \bar{I}_0(\cdot) & \text{on} \quad \Omega, \\ \bar{R}(\cdot, 0) = \bar{R}_0(\cdot) & \text{on} \quad \Omega. \end{cases} \tag{1.2}$$

Here, $\dfrac{\partial}{\partial \eta_{\bar{D}^M}} = \displaystyle\sum_{i,j=1}^{n} \eta_i \bar{D}_{ij}^M \frac{\partial}{\partial x_j}$, where $\eta = (\eta_1, \dots, \eta_n)$ is the unitary exterior normal to Ω at the boundary.

As in Maidana and Yang [5], the mosquito population is divided into subpopulations: winged form (mature female mosquitoes) and aquatic form (comprising eggs, larvae and pupae); their spatial densities are denoted by $\bar{M}(x, \bar{t})$ and $\bar{A}(x, \bar{t})$, respectively. In the winged population we consider the susceptible and infected forms, whose spatial densities are respectively denoted by $\bar{M}_S(x, \bar{t})$ and $\bar{M}_I(x, \bar{t})$, and we obviously have $\bar{M}(x, \bar{t}) = \bar{M}_S(x, \bar{t}) + \bar{M}_I(x, \bar{t})$.

Concerning the human population, again as in Maidana and Yang [5], it is divided into the subpopulations: susceptible, infected and removed (or immune); their respective spatial densities are denoted by $\bar{H}(x, \bar{t})$, $\bar{I}(x, \bar{t})$ and $\bar{R}(x, \bar{t})$; the total human population is $\bar{N}(x, \bar{t}) = \bar{H}(x, \bar{t}) + \bar{I}(x, \bar{t}) + \bar{R}(x, \bar{t})$.