J. Math. Anal. Appl. 442 (2016) 273–290

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Comparison between model equations for long waves and blow-up phenomena

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ARTICLE INFO

Article history:
Received 24 January 2016
Available online 27 April 2016
Submitted by J. Lenells

Keywords:
Nonlinear dispersive wave equations
Water wave models
KdV equation
BBM equation
Cauchy problem
Local & global well-posedness

ABSTRACT

Recently, to describe the unidirectional propagation of water waves, Bona et al. [7] introduced a fifth order KdV–BBM type model

$$\eta_t + \eta_x - \frac{1}{6}\eta_{xxt} + \delta_1\eta_{xxxxt} + \delta_2\eta_{xxxxx} + \frac{3}{4}(\eta^2)_x + \gamma(\eta^2)_{xxx} - \frac{1}{12}(\eta_x^2)_x - \frac{1}{4}(\eta^3)_x = 0, \quad (0.1)$$

where $\eta = \eta(x,t)$ is a real-valued function, and $\delta_1 > 0$, $\delta_2, \gamma \in \mathbb{R}$. In this work, we plan to compare solution of the initial value problem (IVP) associated to the fifth-order KDV–BBM type model (0.1) to that of the IVP associated to the fifth-order KdV model

$$u_t + \delta_3 u_{xxxxx} + c_1 u_x u_{xx} + c_2 u u_{xxx} + c_3 u^2 u_x = 0, \quad (0.2)$$

where $u = u(x,t)$ is a real-valued function and δ_3, c_1, c_2 and c_3 are real constants with $\delta_3 \neq 0$. This later model (0.2) was proposed by Benney in [4] to describe the interaction of long and short waves. Also, we will study the possibility of blow-up phenomenon of the fifth-order KDV–BBM type model under certain restrictions on the coefficients.

1. Introduction

Propagation of waves on the surface of an ideal fluid under gravitational force is governed by the Euler equations. In many practical situations full Euler equations appear more complex than is necessary for the

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http://dx.doi.org/10.1016/j.jmaa.2016.04.062
0022-247X/

undertaken modeling situation. Consequently, in literature several approximate models are derived, among them, Boussinesq equations

$$u_{tt} = u_{xx} + (u^2)_{xx} + u_{xxxx},$$

and its regularized version

$$u_{tt} = u_{xx} + (u^2)_{xx} + u_{xxtt},$$

are the most well known, see [9].

Recently, Bona et al. [9] derived the first-order and the second-order correct Boussinesq systems from the original Euler equations using respectively the first and the second order approximations. The both systems derived in [9] describe the two-way propagation of waves. In practical situations and in numerical purpose, it is more suitable to have a model that governs the one-way propagation. The most famous one way model is the Korteweg–de Vries (KdV) equation

$$u_t + u_{xxx} + uu_x = 0.$$

In particular, it is worth noting that, to obtain an approximate one-way model, in the Boussinesq regime, one generally uses a relation

$$u_x = -u_t + O(\alpha, \beta) \qquad \text{as} \quad \alpha, \beta \to 0, \tag{1.3}$$

where α, β are small parameters related to small amplitude and long wavelength. For instance, if A is a typical amplitude of the wave in the channel with constant depth h and l is a typical wavelength, the conditions of the models are $\alpha = \frac{A}{h}$, $\beta = \frac{h^2}{l^2}$, and the assumption that the Stokes number S be $S = \frac{\alpha}{\beta} \approx 1$.

Using this sort of argument it is possible to derive from the Boussinesq systems, several one way models for long waves as the well known Korteweg–de Vries (KdV) and Benjamin–Bona–Mahony (BBM) equations (see for example [3,6] and references therein).

Quite recently, to describe the unidirectional propagation of water waves, Bona et al. [7] introduced a fifth order KdV–BBM type model

$$\begin{cases} \eta_t + \eta_x - \frac{1}{6}\eta_{xxt} + \delta_1\eta_{xxxxt} + \delta_2\eta_{xxxxx} + \frac{3}{4}(\eta^2)_x + \gamma(\eta^2)_{xxx} - \frac{1}{12}(\eta_x^2)_x - \frac{1}{4}(\eta^3)_x = 0, \\ \eta(,0) = \eta_0(x), \end{cases} \tag{1.4}$$

where $\eta = \eta(x,t)$ is a real-valued function, and $\delta_1 > 0$, $\delta_2, \gamma \in \mathbb{R}$. It was formally obtained as a second order approximation from the higher order generalized Boussinesq system derived by Bona et al. [9].

Note that, if one uses the relation (1.3) to obtain an one-way model like the one in (1.4) from the second order Boussinesq system, there is some loss of information coming from the interacting terms, because that could be not so small. Consequently the resulting equation does not have a correct dispersion relation. Taking this observation into consideration, a correction term is introduced in [7] to obtain a fifth order mathematical model (1.4) describing long waves propagating mainly in one direction. A detailed discussion about derivation and well-posedness theory of this model can be found in [7]. For convenience of the readers we state the following well-posedness result to the IVP (1.4) obtained in [7].

Theorem 1.1. ([7]) *Assume $\delta_1 > 0$. For any $s \geq 1$ and for given $\eta_0 \in H^s(\mathbb{R})$, there exist a time $T = T(\|\eta_0\|_{H^s})$ and a unique solution $\eta \in C([0,T]; H^s)$ to the IVP (1.4) that depends continuously on the initial data.*