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Complex symmetric C_0-semigroups on the Fock space ☆

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A B S T R A C T

In this paper we introduce a new concept in dynamical systems: the complex symmetric operator C_0-semigroups (groups) acting on Hilbert space. Using techniques of weighted composition operators we study these semigroups on the Fock space $\mathcal{F}^2$ and show, in particular, that complex symmetric operator C_0-semigroups contain the familiar unitary semigroups as a proper subclass.

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1. Introduction

1.1. Semigroups of bounded linear operators

The semigroup theory of bounded linear operators arises in many areas of mathematics, for instance: the problem of solving the functional equation $u(t+s) = u(t)u(s)$, the problem of finding the exponential function of a matrix, etc.

In a general setting, given a complex Banach space X with norm $\|\cdot\|$, $\mathcal{L}(X)$ denotes the Banach algebra of all bounded linear operators on X endowed with the operator norm. Consider mappings $T(\cdot)\colon \mathbb{R}_+ \to \mathcal{L}(X)$.

Definition 1.1. A family $(T(s))_{s\geq 0}$ of bounded linear operators on a Banach space X, is called a *semigroup* (or *linear dynamical system*) on X, if

(1) $T(0) = I$, the identity operator on X,
(2) $T(t+s) = T(t)T(s)$, for every $t, s \geq 0$.

These two axioms are algebraic and state that T is a representation of a semigroup $(\mathbb{R}_+, +)$.

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A typical example of a semigroup is the operator-valued exponential function

$$e^{tA} := \sum_{k=0}^{\infty} \frac{t^k A^k}{k!},$$

where $A \in \mathcal{L}(X)$.

Definition 1.2. The family $(T(s))_{s\geq 0}$ is called a *strongly continuous semigroup*, or *C_0-semigroup*, if it is a semigroup and is strongly continuous, that is,

(3) $\lim_{s\to 0^+} T(s)x = x$, for every $x \in X$.

This axiom is topological and states that T is continuous in the strong operator topology.

Note that a semigroup $(T(s))_{s\geq 0}$ on X is strongly continuous if and only if it is a weakly continuous. That is, in the definition above, using the weak topology instead of the strong topology does not change the class of semigroups.

Definition 1.3. The *generator* A: $\mathrm{dom}(A) \subseteq X \to X$ of a C_0-semigroup $(T(s))_{s\geq 0}$ is an operator A defined by

$$Ax = \lim_{s\to 0^+} \frac{T(s)x - x}{s},$$

with the domain of definition

$$\mathrm{dom}(A) = \left\{ x \in X : \lim_{s\to 0^+} \frac{T(s)x - x}{s} \text{ exists} \right\}.$$

We note that a generator of a C_0-semigroup is a closed and densely defined linear operator that determines the semigroup uniquely.

For a bounded linear operator A on X, $\sigma(A)$ and $\sigma_p(A)$ denote respectively the spectrum and the point spectrum of A. Let us recall the spectral relationships between the C_0-semigroup and the generator. The spectral inclusion theorem states that

$$e^{t\sigma(A)} \subseteq \sigma(T(t)), \text{ for all } t \geq 0, \tag{1.1}$$

while the spectral mapping theorem holds for the point spectrum

$$e^{t\sigma_p(A)} = \sigma_p(T(t)) \setminus \{0\}, \text{ for all } t \geq 0. \tag{1.2}$$

Likewise, for $t, s \in \mathbb{R}$, we can "extend" Definitions 1.1–1.3 to *groups* (C_0-groups) and their generators. We refer the reader to the monograph [1] for more detailed information on semigroups (groups) and C_0-semigroups (groups).

1.2. Complex symmetric operators

In analysis, many problems led mathematicians to the study of non-Hermitian operators. Among them, the so-called complex symmetric operators attracted a great deal of attention of operator theorists. Let us recall some notions.

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