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Global boundedness in a quasilinear attraction–repulsion chemotaxis model with nonlinear sensitivity

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ABSTRACT

This paper considers the attraction–repulsion chemotaxis model with homogeneous Neumann boundary conditions in a smooth, bounded, convex domain. In this model, when the scaling constant is zero and the chemotactic sensitivity functions are nonlinear, we prove that this system possesses a unique global classical solution that is uniformly bounded under some assumptions; when the scaling constant is one and one of the chemotactic sensitivity functions is nonlinear, we also obtain a unique bounded global classical solution under some assumptions.

1. Introduction

In this paper, we consider the attraction–repulsion chemotaxis model with nonlinear signal-dependent sensitivity

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u - \nabla \cdot (u\chi(v)\nabla v) + \nabla \cdot (u^r \xi(w)\nabla w), & x \in \Omega,\ t > 0, \\ \tau \dfrac{\partial v}{\partial t} = \Delta v - \alpha v + \beta u, & x \in \Omega,\ t > 0, \\ 0 = \Delta w - \gamma w + \delta u, & x \in \Omega,\ t > 0, \\ \dfrac{\partial u(x,t)}{\partial \nu} = \dfrac{\partial v(x,t)}{\partial \nu} = \dfrac{\partial w(x,t)}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x),\ \ \tau v(x,0) = \tau v_0(x) & x \in \Omega. \end{cases} \tag{1.1}$$

Here $u(x,t)$ represents the density of a cell at location $x \in \Omega$ and time t, $v(x,t)$ denotes the concentration of an attractive signal, $w(x,t)$ is the concentration of a repulsive signal; Ω is a bounded, convex domain in $\mathbb{R}^n$

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($n \geq 1$) with a smooth boundary $\partial\Omega$; the homogeneous Neumann boundary conditions are imposed for u, v and w, so the system is a closed one; the movements are also cell density dependent which are indicated by the functions $\chi(v)$ and $\xi(w)$. Here τ is the nonnegative scaling constant. We assume that the functions $\chi(v)$ and $\xi(w)$ satisfy the following hypotheses:

(H_1) the function $\chi \in C^1((0,\infty))$, $0 < \chi(v) \leq \frac{\chi_0}{v^p}$ with $p \geq 1$ and $\chi_0 > 0$, for all $v > 0$;
(H_2) the function $\xi \in C^1((0,\infty))$, $\xi(w) = \frac{1}{w}\xi_0(w)$, $\xi'(w) \leq 0$, $0 \leq \xi_0(w) \leq \frac{\xi_0}{w^q}$ with $q \geq 1$ and $\xi_0 > 0$, for all $w > 0$;

or

(H_3) the function $\chi(v) = \chi_0$ which is a positive constant;
(H_4) the function $\xi(w) = \frac{\xi_0}{w}$, for all $w > 0$, where ξ_0 is a positive constant;
here χ_0 and ξ_0 are the chemotaxis coefficients, which measure the strength of the attraction and repulsion, respectively.

Chemotaxis is a chemosensitive movement of species which may detect and respond to chemical substances in the environment. The first model about chemotaxis was proposed by Keller and Segel [21]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u - \chi \nabla \cdot (u \nabla v), & x \in \Omega, \\ \dfrac{\partial v}{\partial t} = \Delta v - v + u, & x \in \Omega, \end{cases} \tag{1.2}$$

which describes the aggregation process of the slime mold formation in *Dictyostelium Discoidium*, where v denotes the chemical concentration and u is the concentration of species. For this system, there have been abundant results. Osaki and Yagi [30] found that when $n = 1$, all the solutions are global and bounded. When $n \geq 2$, blow-up may happen[10,14,40]. Furthermore, under some assumptions, when $n \geq 2$, the global existence and boundedness of the solution are also obtained in [28,39]. Global existence, boundedness or blowup of solutions in more general quasilinear parabolic–parabolic chemotaxis systems with nonlinear sensitivity functions and source terms have been studied extensively, see for example, [3,4,16,17,26,35–37, 43]. Various chemotaxis models and mathematical theories of Keller–Segel type models have been surveyed in [1,11–13].

Our work here for the system (1.1) is motivated by recent work on global existence and boundedness in the Keller–Segel systems as follows:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u - \nabla \cdot (u \chi(v) \nabla v), & x \in \Omega, \\ \tau \dfrac{\partial v}{\partial t} = \Delta v - v + u, & x \in \Omega, \end{cases} \tag{1.3}$$

and

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u - \nabla \cdot (u \chi(v) \nabla v) + \nabla \cdot (u \xi(w) \nabla w), & x \in \Omega, \\ \tau_1 \dfrac{\partial v}{\partial t} = \Delta v - \alpha v + \beta u, & x \in \Omega, \\ \tau_2 \dfrac{\partial w}{\partial t} = \Delta w - \gamma w + \delta u, & x \in \Omega. \end{cases} \tag{1.4}$$