J. Math. Anal. Appl. 442 (2016) 600–626

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Finite-time and infinite-time ruin probabilities in a two-dimensional delayed renewal risk model with Sarmanov dependent claims

Yang Yang [a,b,*], Kam C. Yuen [c]

[a] The Key Lab of Financial Engineering of Jiangsu Province, Nanjing Audit University, Nanjing, 211815, China
[b] School of Economics and Management, Southeast University, Nanjing, 210096, China
[c] Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam Road, Hong Kong

ARTICLE INFO

Article history:
Received 7 August 2015
Available online 3 May 2016
Submitted by V. Pozdnyakov

Keywords:
Asymptotics
Two-dimensional delayed renewal risk model
Finite-time and infinite-time ruin probabilities
Bivariate Sarmanov dependence
Subexponentiality
Extended regular variation

ABSTRACT

Consider a two-dimensional delayed renewal risk model with a constant interest rate, where the claim sizes of the two classes form a sequence of independent and identically distributed random vectors following a common bivariate Sarmanov distribution. In the presence of heavy-tailed claim sizes, some asymptotic formulas are derived for the finite-time and infinite-time ruin probabilities.

1. Introduction

In this paper, we address the ruin probabilities of an insurance company in a two-dimensional delayed renewal risk model with a constant interest rate and dependent claims, in which the company operates two lines of business. Each line is assumed to be exposed to catastrophic risks like earthquakes, floods or terrorist attacks. Such risks may affect the two lines of the company at the same time, so the two lines of business share a common claim-number process and some dependence structure may exist between them. For example, one line of business is to pay the total claims of all casualties caused by an earthquake, and another is to pay the total claims of all property losses by the same earthquake. In such a model, the claim sizes of

* Corresponding author at: The Key Lab of Financial Engineering of Jiangsu Province, Nanjing Audit University, Nanjing, 211815, China. Fax: +86 2558318699.
E-mail addresses: yyangmath@gmail.com (Y. Yang), kcyuen@hku.hk (K.C. Yuen).

http://dx.doi.org/10.1016/j.jmaa.2016.04.068
0022-247X/

the two classes $\{\mathbf{X}_i = (X_i^{(1)}, X_i^{(2)})^{\mathrm{T}},\ i \geq 1\}$ form a sequence of independent, identically distributed (i.i.d.) and nonnegative random vectors with a generic random vector $\mathbf{X} = (X^{(1)}, X^{(2)})^{\mathrm{T}}$, whose components are however dependent and have marginal distribution functions (d.f.'s) F_1 and F_2, respectively; and the claim inter-arrival times $\{\theta_i,\ i \geq 1\}$, independent of $\{\mathbf{X}_i,\ i \geq 1\}$, are another sequence of independent, nonnegative and nondegenerate random variables (r.v.'s). If $\{\theta_i,\ i \geq 2\}$ are identically distributed with common d.f. G, and θ_1 has an arbitrary d.f. G_1, which need not be equal to G (one may have (partial) information on the process before time 0), then the successive claim arrival times, denoted by $\{\tau_n = \sum_{i=1}^n \theta_i,\ n \geq 1\}$, constitute a delayed renewal counting process

$$N(t) = \sup\left\{n : \sum_{i=1}^{n} \theta_i \leq t\right\} = \sum_{n=1}^{\infty} \mathbb{1}_{\{\tau_n \leq t\}},\ t \geq 0, \tag{1.1}$$

with a finite mean function $\lambda(t) = \mathbb{E}N(t) = \sum_{i=1}^{\infty} \mathbb{P}(\tau_i \leq t)$, where $\mathbb{1}_A$ denotes the indicator function of an event A. If θ_1 is identically distributed as θ_i, $i \geq 2$ (i.e. $G_1 = G$), then the model is reduced to a two-dimensional (zero-delayed) renewal risk model. The vector of the total premium accumulated up to time $t \geq 0$, denoted by $\mathbf{C}(t) = (C_1(t), C_2(t))^{\mathrm{T}}$ with $\mathbf{C}(0) = (0, 0)^{\mathrm{T}}$ and $C_i(t) < \infty$, $i = 1, 2$, almost surely for every $t > 0$, is a nonnegative and nondecreasing two-dimensional stochastic process. Assume that $\{\mathbf{X}_i,\ i \geq 1\}$, $\{\theta_i,\ i \geq 1\}$ and $\{\mathbf{C}(t),\ t \geq 0\}$ are mutually independent. Let $\delta \geq 0$ be the constant interest rate; that is to say, from time 0 to time t one dollar accumulates $e^{\delta t}$ dollars, and $\mathbf{x} = (x_1, x_2)^{\mathrm{T}}$ denotes the initial surplus vector. In this two-dimensional setting, the discounted surplus process up to time $t \geq 0$, denoted by $\mathbf{D}(t) = (D_1(t), D_2(t))^{\mathrm{T}}$ has the form

$$\mathbf{D}(t) = \mathbf{x} + \int_{0-}^{t} e^{-\delta s}\mathbf{C}(\mathrm{d}s) - \sum_{i=1}^{N(t)} \mathbf{X}_i e^{-\delta \tau_i}. \tag{1.2}$$

In the one-dimensional setting, the finite-time and infinite-time ruin probabilities are defined, respectively, as, for some finite $T > 0$,

$$\psi(x_1; T) = \mathbb{P}(D_1(t) < 0 \text{ for some } t \in [0, T] | D_1(0) = x_1)$$

and

$$\psi(x_1; \infty) = \mathbb{P}(D_1(t) < 0 \text{ for some } t \in [0, \infty) | D_1(0) = x_1).$$

However, in the two-dimensional case, there are several definitions of ruin; for example, see Cai and Li [2,3].

In this paper, we investigate the ruin probabilities by means of the ruin time

$$T_{\max} = \inf\{t > 0 : \max\{D_1(t), D_2(t)\} < 0\},$$

where by convention, $\inf \emptyset = \infty$. This was also investigated by Li et al. [13]. Then, the finite-time ruin probability within a finite time $t > 0$ and the infinite-time ruin probability can be defined, respectively, as

$$\psi(\mathbf{x}; t) = \mathbb{P}(T_{\max} \leq t | \mathbf{D}(0) = \mathbf{x}) \tag{1.3}$$

and

$$\psi(\mathbf{x}; \infty) = \mathbb{P}(T_{\max} < \infty | \mathbf{D}(0) = \mathbf{x}). \tag{1.4}$$