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A class of evolution variational inequalities with memory and its application to viscoelastic frictional contact problems

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ABSTRACT

In this paper we consider the first order evolution variational inequality involving two history-dependent operators. A result on existence and uniqueness of solution is proved. We illustrate the applicability of this result by considering a dynamic frictional contact problem for viscoelastic material with the normal compliance contact condition with memory and Coulomb's law of dry friction.

1. Introduction

In this paper we investigate the problem of existence and uniqueness of solution to the Cauchy problem for the first order evolution variational inequality. We formulate this problem in the framework of evolution triple of spaces. The variational inequality involves a nonlinear strongly monotone operator and a convex potential which depends on a nonlinear history-dependent operator. These properties represent the main novelty of the problem. We underline that such class of abstract evolution variational inequalities, to the best of our knowledge, has not been studied in the literature till now. The idea of the proof of our existence result, Theorem 5, is based on a result on existence of solution to evolution subdifferential inclusions based on a surjectivity result and provided recently by Migórski et al. [26], and a fixed point argument introduced and developed by Han and Sofonea [14], Shillor et al. [30], and Sofonea and Matei [35].

The classical material on variational inequalities can be found in well known monographs by Baiocchi and Capelo [2], Brezis [3], Duvaut and Lions [9], Kikuchi and Oden [16], Kinderlehrer and Stampacchia [17],

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among others. We remark that for the parabolic variational inequalities there are other approaches to establish existence of solution: the translation method which exploits the fact that the operator $-d/dt$ generates the semigroup of translations (cf. Showalter [31]), the Rothe method (method of lines) which extends the backward Euler scheme (see Kačur [15]), and the penalty method which provides the approximation problem that penalizes the admissible functions for violating a constraint (cf. Brezis [3], Kikuchi and Oden [16], Lions [20]). We refer to Han and Sofonea [14], Rudd and Schmitt [29], Sofonea and Matei [35] and the references therein for more discussion on these methods. More recently, various classes of variational inequalities with history-dependent operators were studied and applied to quasistatic contact problems in Cheng et al. [6], Farcaş et al. [10], Sofonea et al. [32,34,37,38]. Note also that related results on other classes of history-dependent stationary hemivariational inequalities and variational-hemivariational inequalities and their applications can be found in Han et al. [11,13], Han et al. [12], Migórski [21], Migórski et al. [22–25], Ogorzały [27], Sofonea et al. [33] and the references therein.

The motivation for our study comes from several problems arising in mechanics, physics and engineering science which have as variational formulations the evolution variational inequalities with memory terms. In the present paper, our main result on abstract evolution variational inequality with history-dependent operator is illustrated by a mathematical model which describes the dynamic frictional contact between a body and a foundation and to which our result can be applied. In the contact model, the material of the body, which occupies a bounded domain Ω, is described by a fully nonlinear viscoelastic law with long memory of the form

$$\boldsymbol{\sigma}(t) = \mathcal{A}(t, \boldsymbol{\varepsilon}(\boldsymbol{u}'(t))) + \mathcal{B}(t, \boldsymbol{\varepsilon}(\boldsymbol{u}(t))) + \int_0^t \mathcal{K}(t-s, \boldsymbol{\varepsilon}(\boldsymbol{u}'(s)))\, ds \quad \text{in } \Omega \times (0,T), \tag{1}$$

where $(0,T)$ denotes the finite time interval and the nonlinear operators $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{K}$ describe the viscous, elastic and relaxation properties of the material, respectively. When $\mathcal{K}$ vanishes, then (1) reduces to the well known Kelvin–Voigt constitutive law extensively studied in the literature, see Han and Sofonea [14], Shillor et al. [30], Sofonea and Matei [35] and the references therein. On the other hand, the contact is modeled by the following condition

$$-\sigma_\nu(t) = k\Big(\int_0^t u_\nu(s)\, ds\Big)\, p(t, u_\nu(t)) + \int_0^t b(t-s, u_\nu(s))\, ds \quad \text{on } \Gamma_C \times (0,T). \tag{2}$$

Here k represents the stiffness coefficient, p is a contact function, b is a memory function, and Γ_C denotes the contact surface. Dynamic contact problems for viscoelastic and viscoplastic materials with the contact law (2) and a nonmonotone friction condition were studied in Sofonea et al. [36] with $k = 1$ and with $b = 0$, respectively. Condition (2) with $k = 1$ and $b(x,t,r) = \widetilde{b}(t,s)r_+$ was considered in Sofonea and Farcaş [32] for the quasistatic frictional contact problem. Note that if k is a positive constant and b vanishes, then condition (2) was treated for various choices of contact function p in Han and Sofonea [14], Anderson [1], Kikuchi and Oden [16], Klarbring et al. [18], Rochdi et al. [28].

In the present paper, the contact condition (2) in the model is supplemented by the friction condition which includes the Coulomb law of dry friction with friction bound $F_b = F_b(\sigma_\nu) = \mu|\sigma_\nu|$, where μ is the coefficient of friction. We derive a variational formulation of the dynamic contact problem which turn to be a second order in time variational inequality for the displacement field or, equivalently, a first order in time variational inequality with history term for the velocity. For the latter we provide a result, Theorem 7, on a unique solvability and regularity which holds under a smallness condition and which is proved by exploiting our abstract result. As far as we know there is no existence and uniqueness result for this problem in the literature. Related results on dynamic contact problems modeled by the normal compliance condition can be also found in Campo et al. [4] for frictionless contact and in Chau et al. [5] for frictional contact.