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Attractors for second order lattice systems with almost periodic symbols in weighted spaces

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ABSTRACT

We study the existence of a uniform global attractor for a family of second order non-autonomous lattice dynamical systems with almost periodic symbols in a suitable weighted space of infinite sequences. The assumptions on the nonlinear part present the main difficulty of this work. Lemmas 2 and 4 are helpful to overcome such a difficulty. A uniform absorbing set O and uniform estimates on the tails of solutions with respect to initial data from O and all translations of the external term involved are introduced. Such uniform estimates are needed to obtain the asymptotic compactness of solution operators which is a major step towards proving the existence of attractors.

1. Introduction

Lattice dynamical systems (LDSs) are infinite systems of ordinary differential equations or of difference equations, indexed by points in a lattice such as the m-dimensional integer lattice $\mathbb{Z}^m$. They arise in different fields of applications in science and engineering, for instance, in propagation of nerve pulses in myelinated axons, electrical engineering, pattern recognition, image processing, chemical reaction theory, etc. In each case, they have their own form, but in some cases, they appear as spatial discretizations of continuous partial differential equations (PDEs) [8,13].

The global attractor is the natural object to study the long time behavior of a given dynamical system since it is the smallest compact set, with respect to inclusion, that is invariant, attracts all the trajectories originated from the whole phase space and sometimes it has finite dimension. For continuous PDEs, it is difficult to study the existence of global attractors on unbounded domains because some compactness properties are not available. Therefore it is significant to study the existence of global attractors for LDSs because of the importance of such systems, and they can be considered as an approximation to the corresponding continuous PDEs if they arise as spatial discretizations for such PDEs.

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Chepyzhov and Vishik [12,11] introduced a simple method to investigate the existence of a uniform attractor for the family of processes associated with non-autonomous infinite dimensional dynamical systems of the form

$$\frac{du}{dt} = G_{\sigma(t)}(u), u(\tau) = u_\tau, \quad t \geq \tau, \tau \in \mathbb{R},$$

where u belongs to a Banach space E and σ belongs to an appropriate symbol space Σ.

Recently, the existence of global attractor, uniform attractor, pullback attractor, and random attractor for different types of autonomous, non-autonomous, and stochastic LDSs in standard and weighted spaces of infinite lattices has been carefully investigated [1–4,7,6,14,15,17,18,21–25].

In [25], S. Zhou and M. Zhao studied the existence of uniform global and exponential attractors in a suitable weighted space for a second order non-autonomous LDS with quasi-periodic external forces of the following form:

$$\ddot{u}_i + \alpha \dot{u}_i + (Au)_i + \lambda_i u_i + f_i(u_i) = a_i h_i(\tilde{\sigma}(t)), \quad i \in \mathbb{Z}, t > \tau, \tau \in \mathbb{R}.$$

Here we study the existence of a uniform global attractor in a suitable weighted space for a new family of second order non-autonomous LDSs with almost periodic symbols which is related to the following LDS:

$$\ddot{u}_i + \alpha \dot{u}_i + (Au)_i + g_i(u_i, t) = 0, \quad i \in \mathbb{Z}, t > \tau, \tau \in \mathbb{R}, \tag{1.1}$$

with initial data

$$u_i(\tau) = u_{0i,\tau}, \dot{u}_i(\tau) = u_{1i,\tau}, \quad i \in \mathbb{Z}, \tag{1.2}$$

where a uniform absorbing set is presented and uniform estimates on the tails of solutions are introduced. Note that for a family of non-autonomous LDSs, a family of processes appears instead of a semigroup. In such a case, the estimates on the tails of the solutions must be uniform with respect to initial data from a bounded set as well as all translations of the external term involved in the system. These uniform estimates are crucial to obtaining the asymptotic compactness of solution semigroup associated with such a system which is a major step towards proving the existence of uniform global attractors.

2. Preliminaries

Let $\rho : \mathbb{Z} \to (0, +\infty)$, $i \to \rho(i) = \rho_i$, be a weight function such that for some positive constants c_0, c_1, and c_2, we have

$$0 < \rho(i) \leq c_0 < \infty, \quad i \in \mathbb{Z}, \tag{2.1}$$

$$\rho(i \pm 1) \leq c_1 \rho(i), \text{ and } |\rho(i \pm 1) - \rho(i)| \leq c_2 \rho(i), \quad i \in \mathbb{Z}. \tag{2.2}$$

It is clear that c_1 can not be smaller than 1. Typical examples of such weight functions are $\rho(x) = (1 + \epsilon^2 x^2)^{-a}$, $a > 1/2$, and $\rho(x) = e^{-\epsilon|x|}$, $x \in \mathbb{Z}$, where $\epsilon > 0$.

Consider the Hilbert space

$$l_\rho^2 = \left\{ u = (u_i)_{i\in\mathbb{Z}} : \sum_{i\in\mathbb{Z}} \rho_i u_i^2 < \infty, \quad u_i \in \mathbb{R} \right\}, \tag{2.3}$$

whose inner product and norm are given by:

$$\langle u, v \rangle_\rho = \sum_{i\in\mathbb{Z}} \rho_i u_i v_i, \|u\|_\rho = \langle u, u \rangle_\rho^{1/2}, \quad u = (u_i)_{i\in\mathbb{Z}}, v = (v_i)_{i\in\mathbb{Z}} \in l_\rho^2. \tag{2.4}$$