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Traveling wave solutions for a continuous and discrete diffusive predator–prey model

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ABSTRACT

We study a diffusive predator–prey model of Lotka–Volterra type functional response in which both species obey the logistic growth such that the carrying capacity of the predator is proportional to the prey population and the one for prey is a constant. Both continuous and discrete diffusion are addressed. Our aim is to see whether both species can survive eventually, if an alien invading predator is introduced to the habitat of an existing prey. The answer to this question is positive under certain restriction on the parameter. Applying Schauder's fixed point theory with the help of suitable upper and lower solutions, the existence of traveling wave solutions for this model is proven. Furthermore, by deriving the non-existence of traveling wave solutions, we also determine the minimal speed of traveling waves for this model. This provides an estimation of the invasion speed.

1. Introduction

In this paper, we consider the following diffusive predator–prey model

$$\begin{cases} u_t = u_{xx} + ru(1-u) - rkuv, \\ v_t = dv_{xx} + sv\left(1 - \dfrac{v}{u}\right), \end{cases} \tag{1.1}$$

where the unknown functions u, v of (x, t), $x, t \in \mathbb{R}$, stand for the population densities of prey and predator species at position x and time t, respectively, d, r, s, k are positive constants such that $1, d$ are diffusion coefficients and r, s are intrinsic growth rates of species u, v, respectively. The functional response of predator to prey is given by rku, which is of *Lotka–Volterra* type. The prey obeys the logistic growth and its carrying capacity is normalized to be 1. However, the density of predator follows a logistic dynamics with a varying carrying capacity proportional to the density of prey.

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In fact, the model (1.1) is a special case of the following Holling–Tanner type predator–prey model (cf. [24,25]):

$$\begin{cases} u_t = u_{xx} + ru(1-u) - \dfrac{rku}{a+bu}v, \\ v_t = dv_{xx} + sv\left(1 - \dfrac{v}{u}\right), \end{cases}$$

when $a = 1, b = 0$. For the case when $a = 0, b = 1$, it is possible that the density of prey may vanish so that quenching or extinction phenomenon may occur. For this singular behavior, we refer the reader to [5,4,7,8,10] and the references cited therein.

It is easy to see that (1.1) has two constant steady states $(1, 0)$ and $(1/(1 + k), 1/(1 + k))$. In [6], the authors studied the model (1.1) in a bounded domain with zero Neumann boundary condition. Among other things, by constructing a delicate Lyapunov function, they show that the unique positive constant state $(1/(1 + k), 1/(1 + k))$ is globally stable under certain restrictions on k. In other words, this constant state attracts every positive solution of (1.1) for the Neumann initial boundary value problem in a bounded domain. Since the predator will extinct if the prey vanish, the possibility of co-existence is very important from the ecological point view. For the case $a = 1, b > 0$, we refer the reader to [12–14].

In this paper, we consider the case when the habitat is the whole real line. We are interested in the question whether both species can survive eventually, if an alien predator is introduced into the habitat where a prey has been living there. In fact, this question is equivalent to whether the solution of (1.1) tends to the unique positive constant steady state as the time approaches infinity. Therefore, we study the so-called traveling wave solution defined as follows.

A solution of (1.1) is called a traveling wave with speed c if there exist positive functions ϕ_1 and ϕ_2 defined on $\mathbb{R}$ such that $u(x, t) = \phi_1(x + ct)$ and $v(x, t) = \phi_2(x + ct)$. Here ϕ_1 and ϕ_2 are the wave profiles. Set $z := x + ct$ and substitute $(u, v)(x, t) = (\phi_1, \phi_2)(z)$ into (1.1). Then the wave profile (ϕ_1, ϕ_2) satisfies the following system of equations:

$$\begin{cases} \phi_1''(z) - c\phi_1'(z) + r\phi_1(z)[1 - \phi_1(z) - k\phi_2(z)] = 0,\ z \in \mathbb{R}, \\ d\phi_2''(z) - c\phi_2'(z) + s\phi_2(z)\left[1 - \dfrac{\phi_2(z)}{\phi_1(z)}\right] = 0,\ z \in \mathbb{R}. \end{cases} \tag{1.2}$$

Here the prime denotes the derivative with respect to z. As described above, we are interested in the traveling wave solutions of (1.1) connecting $(1, 0)$ and $(1/(1 + k), 1/(1 + k))$. This implies that (ϕ_1, ϕ_2) satisfies the following asymptotic boundary conditions

$$\lim_{z\to-\infty}(\phi_1, \phi_2)(z) = (1, 0), \quad \lim_{z\to+\infty}(\phi_1, \phi_2)(z) = \left(\frac{1}{1+k}, \frac{1}{1+k}\right). \tag{1.3}$$

Note that the existence of such traveling wave solutions (with $c > 0$) means the successful invasion of the predator.

Biologically, it is also interesting to study the invasion speed. A constant c^* is called the minimal speed of traveling waves, if there is a traveling wave of speed c for any $c \geq c^*$ and no wave of speed c exists for $c < c^*$. The minimal speed of traveling waves plays an important role in the estimation of the invasion speed. We prove that the minimal speed of traveling wave solutions of (1.1) is given by $c^* := 2\sqrt{ds}$. Notice that this minimal speed is independent of the parameters r and k.

In this paper, we also consider the following lattice dynamical system (LDS)

$$\begin{cases} \dfrac{du_i}{dt} = (u_{i+1} + u_{i-1} - 2u_i) + ru_i(1 - u_i - kv_i),\ i \in \mathbb{Z}, \\ \dfrac{dv_i}{dt} = d(v_{i+1} + v_{i-1} - 2v_i) + sv_i\left(1 - \dfrac{v_i}{u_i}\right),\ i \in \mathbb{Z}, \end{cases} \tag{1.4}$$