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An accelerated technique for solving a coupled system of differential equations for a catalytic converter in interphase heat transfer

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A B S T R A C T

This paper deals with an accelerated iterative procedure for a coupled system of partial differential equations arising from a catalytic converter model. Based on the main theorem, a finite difference based numerical method is developed. The monotone property as well as the convergence analysis and the error estimate of the proposed finite difference scheme are proved theoretically. The efficiency of the proposed scheme is illustrated by providing a comparative numerical study with the existing method. The proposed iterative procedure reduces the number of iterations at least by forty percent for certain benchmarks available in the literature.

1. Introduction

Catalytic converter is a reliable emissions control device that converts toxic pollutants in exhaust gas to less toxic pollutants and is located in the exhaust system of automobiles. The increasing concern about the atmospheric pollution caused due to the harmful emissions from the vehicles leads to the development of various mathematical models for the study of interphase heat-transfer problem in catalytic converter [3–6,8,10,12,13]. One of such models is studied in [7] where the vehicle and converter temperatures are governed by a coupled system of a first order partial differential equation and an ordinary differential equation. After suitable simplifications [1,2,9], the problem reduces to the following system.

$$\begin{cases} \frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} + cu = cv, \quad t > 0, 0 < x \le l \\ \frac{\partial v}{\partial t} + bv = bu + \lambda \exp(v), \quad t > 0, 0 < x \le l \\ u(0,t) = \eta,\ u(x,0) = u_0(x),\ v(x,0) = v_0(x),\ t > 0, 0 < x \le l. \end{cases} \tag{1.1}$$

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The existence and uniqueness of the classical solution of the above coupled system has been proved using the contraction principle in [1]. Later by coupling successive iteration and monotone method, the existence and uniqueness as well as the blowup property of the solution have been discussed in [2]. Based on the main theorem in [2], a finite difference based iterative procedure has been developed in [9] to solve the coupled system numerically. The study in [9] has also proved that the finite difference scheme preserves the monotone property. It is important to note that for the numerical method in [9] based on the successive approximation discussed in [2], the performance of the numerical scheme is slow.

In this paper to accelerate the iterative procedure, a modification to the iterative scheme in [2] is proposed. More specifically, by combining successive iteration and quasilinearization together with monotone method, an accelerated iterative procedure is proposed. The first part of the paper discusses about the convergence analysis, error estimate as well as the monotone property of the proposed accelerated iterative procedure for the continuous case. In the second part, based on this iterative procedure, a new iterative scheme based on finite difference method is proposed to solve the coupled system numerically. This part also proves the convergence and the monotone property of the discretized version of the iterative procedure. Moreover, a detailed error estimate is also derived.

In the proposed iterative scheme, at each step one has to solve a system with variable coefficients distinct from [2] and [9] where constant coefficients are only dealt with. Consequently in the discretized case, a new comparison theorem is developed to obtain the monotone property of the sequences.

This paper is organised as follows. Section 2 provides certain basic results that are used in the following sections. In Section 3, the existence and uniqueness of the coupled system (1.1) is proved via the new accelerated iterative scheme. This section also provides the error estimate for the iterative procedure. Section 4 gives the convergence analysis as well as the error estimate for the proposed numerical scheme. The convergence of the finite difference solution to the continuous solution as the mesh sizes tend to zero is obtained in Section 5. Some numerical results are given in Section 6 to illustrate the efficiency of the proposed scheme. A comparative study is also provided in this section.

2. Preliminaries

In this section, some basic results are stated that will be used to obtain the results in the following sections.

Definition 2.1. [11] An $n \times n$ real matrix $A = (a_{i,j})$ is said to be a $\mathbb{Z}$-matrix if $a_{i,j} \leq 0$ for all $i \neq j$; $1 \leq i,j \leq n$. An $n \times n$ matrix A that can be expressed in the form $A = sI - B$ where $B = (b_{i,j})$ with $b_{i,j} \geq 0$ for all $1 \leq i,j \leq n$ and $s \geq \rho(B)$, the maximum of the moduli of the eigenvalues of B is called an $\mathbb{M}$-matrix.

Note: If A is an $n \times n$ real $\mathbb{Z}$-matrix, then the following statements are equivalent to A being a nonsingular $\mathbb{M}$-matrix.

- All the principal minors of A are positive.
- A is inverse positive; A^{-1} exists and is positive.
- A is monotone; $Ax \geq 0 \Rightarrow x \geq 0$.

For more details on $\mathbb{M}$-matrix, one can refer to [11]. The existence and uniqueness theorem for (1.1) using contraction principle discussed in [1] can be stated as follows

Theorem 2.1. *(Theorem 6; [1]) Suppose $u_0(x) = u(x,0) \in C^1[0,l]$ and $v_0(x) = v(x,0) \in C^1[0,l]$ with $u_0(0) = \eta$. There is a constant $t_{max} > 0$ such that $[0, t_{max})$ is the maximal time interval for the unique solution (u,v) of the differential equation (1.1) on the interval $[0,l] \times [0, t_{max})$.*