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L^p-strong convergence of the averaging principle for slow–fast SPDEs with jumps ☆

Jie Xu

College of Mathematics and Information Science, and Henan Engineering Laboratory for Big Data Statistical Analysis and Optimal Control, Henan Normal University, Xinxiang, Henan 453007, P.R. China

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ABSTRACT

The averaging principle is an important method to extract effective macroscopic dynamic from complex systems with slow component and fast component. This paper concerns the L^p-strong convergence of the averaging principle for two-time-scales stochastic partial differential equations (SPDEs) driven by Wiener processes and Poisson jumps. To achieve this, a key step is to show the existence for an invariant measure with exponentially ergodic property for the fast equation, where the dissipative conditions are needed. Furthermore, it is shown that under suitable assumptions the slow component L^p-strongly converges to the solution of the averaged equation. The rate of the convergence is also obtained as a byproduct.

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1. Introduction

The averaging principle has received significant attention from many practical domains, such as mechanics, molecular dynamic, material science, automatic control and many other areas. The theory of the averaging principle for deterministic systems has been extensively developed. It was firstly investigated by Bogoliubov [7], then by Gikhman [25], Volosov [62] and Besjes [6] for non-linear ordinary differential equations. After that, the theory of averaging was developed by Khasminskii [33] using probabilistic methods for a class of second order parabolic partial differential equations. Since then, there is an extensive literature on averaging principles for parabolic PDEs (cf. [5,31,49,66], etc.). Kouritzin in [39] firstly put forward to dealing with the asymptotic behavior of fundamental solution to the Cauchy initial value problem for singularly perturbed linear parabolic equations of arbitrary order, which explored a point-wise estimate for the difference between the fundamental solution of the original equation and solution of the averaged equation. In [48], Matthies studied the theory of averaging for parabolic PDEs with a rapid time periodic forcing.

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E-mail address: xujiescu@163.com.

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The theory of averaging was firstly considered by Khasminskii [34] for stochastic ordinary equations with different time-scales. The results showed that a stochastic averaging principle occurs for the slow component in a weak sense. Since then, people further made a lot of research work (cf. [19,26,27,44,59,63], etc.). Taking into account the generalized and refined results, it is worth to mention the paper by Veretennikov [60,61], the work of Freidlin and Wentzell [20,21] with notably extensions to convergence in probability. The mean-square type convergence was treated by Golec and Ladde [30] and Givon and co-workers [28]. For the strong convergence, we may refer to [29,32,35–37,40,41,46] for recent related work on averaging for stochastic differential equations in finite dimensional space.

In the past three decades, there are numerous results about the averaging principle for SPDEs (cf. [8,11–14,22–24,42,47,55,64], etc.). All these results are concentrated on semi-linear SPDEs driven by Wiener process without jumps, i.e., the second term is linear and at least one equation driven by Wiener process without jumps, and their proofs are mainly based on some necessary moment estimates, exponential ergodicity and Markov property. Recently, Cerrai [12] presented an averaged principle for slow–fast stochastic reaction–diffusion equations driven by Wiener processes. To get exponential ergodicity, the proof of [12] needed some very strong assumption conditions. In addition, Fu, Liu and Duan [22,23] proved an averaging principle for slow–fast SPDEs driven by Wiener processes without jumps and proposed an explicit rate of convergence. To get some necessary moment estimates and exponential ergodicity, it should be pointed out that the proof of their paper [22,23] depended on the boundedness of the diffusion coefficients in fast equations. Very recently, Fu, Wan, Wang and Liu [24] showed a L^2-strong convergence rate of the averaging principle for stochastic FitzHugh–Nagumo system with two time-scales. Due to the characteristics of the stochastic FitzHugh–Nagumo system, it seems not easy to obtain $L^p(p > 2)$-strong convergence in [24].

On the other hand, the existence and uniqueness of SPDEs with jumps have already been studied in various literature prior to the averaging principle for SPDEs with jumps (cf. [1–3,9,10,18,38,43,45,52,54,57,58,65], etc.). In the theory of SPDEs with jumps, there exist two main tools: the semigroup and the variational methods (or monotone method). One of the merits of semigroup method is that the noise can take values in a large space (cf. [52]). But, it can only deal with semi-linear SPDEs with jumps. The variation method combined with Galerkin's approximation is usually used in the framework of evolution triple (cf. [45]). Thus, as in the deterministic case (cf. [56]), it can tackle a large class of SPDEs with jumps.

However, there are few results on the averaging principle for SPDEs with jumps in infinite dimensional space. Our aim in the present paper is to prove the L^p-strong convergence of the averaging principle for SPDEs with jumps by using the moment estimates method (also called Hasminskii technique or Khasminskii technique) as done in [34]. Thus, the main point is to prove some necessary moment estimates and exponential ergodicity of SPDEs with jumps. Some moment estimates will be realized by stochastic tools under suitable assumptions, for example, Itô's formula, Burkholder–Davis–Gundy inequality, energy formula and Hölder's inequality, etc. Moreover, in order to treat the exponential ergodicity of SPDEs with jumps, we will work in the framework of [14], which is a little different from [4].

The main novelties of this paper are twofold. The first one is to extend the result of [22,23] to jumps case. Concretely speaking, the result here is a generalization of the theorem in [22,23], when we take $c = 0, h = 0$ and the boundedness of g case in this paper. According to the Lévy–Itô decomposition theorem, we discuss the system including a large class of equations driven by $\mathbb{R}$-valued Lévy noise. Secondly, we propose a sufficient condition to get the averaging principle for SPDEs with jumps in infinite dimensional space, where the sufficient condition is not covered by [22,23]. It is the fact that as mentioned above their proofs in [22,23] needed to depend the boundedness of the diffusion terms in fast equations, while the sufficient condition in this paper can not be covered by the boundedness assumption of the diffusion terms in fast equations [22,23].

This paper is organized as follows. In Section 2 we present the framework and some preliminary results. Section 3 deals with the exponential ergodicity of the fast equation with frozen slow component. In Section 4, some a priori estimates are presented. These results will be utilized in the subsequent discussion. Lastly, in

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