J. Math. Anal. Appl. 445 (2017) 394–406

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Analyticity and near optimal time boundary controllability for the linear Klein–Gordon equation

R.S.O. Nunes [a,*], W.D. Bastos [b,1]

[a] *UFMT – Federal University of Mato Grosso, ICET, Department of Mathematics, 78060-900, Cuiabá, MT, Brazil*
[b] *UNESP – São Paulo State University, IBILCE, Department of Mathematics, 15054-000, S.J. Rio Preto, SP, Brazil*

ARTICLE INFO

Article history:
Received 8 January 2015
Available online 5 August 2016
Submitted by H. Zwart

Keywords:
Klein–Gordon equation
Time analyticity
Optimal time boundary controllability
Exact controllability

ABSTRACT

We study time analyticity for the solutions of the linear Klein–Gordon equation in $\mathbb{R}^N$, $N \geq 1$ and use it to obtain essentially optimal time boundary controllability in piecewise smooth domains. The initial state has finite energy and the control is square integrable.

1. Introduction

Let $N \geq 1$ be a positive integer and $\Omega \subset \mathbb{R}^N$ be a bounded domain with piecewise smooth boundary Γ. We denote by η the outward unit normal vector to Γ. Let $c \geq 0$ be a real number and define

$$L = \frac{\partial^2}{\partial t^2} - \Delta + c^2, \qquad \triangle = \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2}.$$

Let $H^m(\Omega)$, $m = 0, 1$ denote the usual Sobolev spaces (see [2]). Using the method of controllability presented in [7] it was shown in [6] that if Γ is a piecewise smooth with no cuspidal points then there exists $T > diam(\Omega)$, sufficient large, such that for any $(u_0, u_1) \in H^1(\Omega) \times L^2(\Omega)$ there exists a control h in $L^2(\Gamma \times [0, T])$ which makes the solution of the problem

$$Lu = 0 \qquad in \qquad \Omega \times [0, T], \tag{1}$$

* Corresponding author.
E-mail addresses: ruiksonsillas@hotmail.com (R.S.O. Nunes), waldemar@ibilce.unesp.br (W.D. Bastos).

1 Supported by grant # 2014/09900-7, São Paulo Research Foundation (FAPESP).

http://dx.doi.org/10.1016/j.jmaa.2016.08.001

$$u(.,0) = u_0, \quad u_t(.,0) = u_1 \qquad in \qquad \Omega, \tag{2}$$

$$\frac{\partial u}{\partial \eta} = h \qquad in \qquad \Gamma \times [0,T], \tag{3}$$

to satisfy the final condition

$$u(.,T) = 0 = u_t(.,T) \quad in \quad \Omega. \tag{4}$$

By making use of a trace theorem presented in [8,1] it was possible to guarantee that the control function h is square integrable on surface $\Gamma \times [0,T]$. We remark that in [6] it was not possible to specify how great is T, but in applications it is important to estimate it. In the present work the main goal is to prove that the time T in the control problem (1)–(4) may be reduced to any value $T > diam(\Omega)$. This is achieved by showing that for the solution u of the Cauchy problem

$$Lu = 0 \qquad in \qquad \mathbb{R}^N \times \mathbb{R}, \tag{5}$$

$$u(.,0) = u_0, \quad u_t(.,0) = u_1 \qquad in \qquad \mathbb{R}^N, \tag{6}$$

the mapping $t \mapsto (u(.,t), u_t(.,t))$ has analytic extension to a region of the complex plane. This argument has been used in [5] for the even dimensional wave equation with smooth initial data. In that paper the author extended the mapping $t \mapsto (u(.,t), u_t(.,t))$ analytically to the complex sector $\Sigma_0 = \{\zeta = T_0 + z, \ |arg(z)| \leq \pi/4\}$, where T_0 is any constant greater than the diameter of a bounded domain which contains the support of the initial data.

The main results of the present paper are the following two theorems.

Theorem 1.1. *Let $U \subset \mathbb{R}^N$, $N \geq 1$, be a bounded domain and $T_0 > diam(U)$ a real number. If $u(.,t)$ is the solution of the Cauchy problem (5)–(6) with initial state $(u(.,0), u_t(.,0)) \in H^1(\mathbb{R}^N) \times L^2(\mathbb{R}^N)$ compactly supported in U then the map $t \mapsto (u(.,t), u_t(.,t))$ extends analytically to the sector $\Sigma_0 = \{\zeta = T_0 + z$, $|arg(z)| \leq \pi/4\}$.*

Theorem 1.2. *Let Ω be a bounded domain of $\mathbb{R}^N$, $N \geq 1$ with boundary Γ piecewise smooth with no cuspidal points. We also assume that Ω is on the same side of its boundary, case $N = 1$ we just require Ω to be a bounded interval. For any $T > diam(\Omega)$, given $(u_0, u_1) \in H^1(\Omega) \times L^2(\Omega)$ there exists a control function $h \in L^2(\Gamma \times [0,T])$ such that the solution of (1)–(3) satisfies the final state (4).*

To prove the Theorem 1.2 we use the Theorem 1.1, local energy decay ([6], Theorem 2.1) and an alternative theorem due to F.V. Atkinson (see, [4], p. 370) to reduce the time of control T to values near $diam(\Omega)$.

The rest of this paper is organized as follows. The Section 2 is devoted to developing the necessary tools to prove the main results. Section 3 is devoted to prove the Theorem 1.1 and in Section 4 we prove the Theorem 1.2.

2. Preliminaries

Let $U \subset \mathbb{R}^N$, $N \geq 1$ be a bounded domain and $(u_0, u_1) \in H^1(\mathbb{R}^N) \times L^2(\mathbb{R}^N)$ initial data supported in U. In [6] it was established that for $t > diam(U)$ and $x \in U$ the solution of the Cauchy problem (5)–(6) is given explicitly by the formulas bellow: