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On the porous-elastic system with Kelvin–Voigt damping

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ABSTRACT

In this article we are considering the one-dimensional equations of a homogeneous and isotropic porous elastic solid with Kelvin–Voigt damping. We prove that the semigroup associated with the system (1.3) with Dirichlet–Dirichlet boundary conditions or Dirichlet–Neumann boundary conditions is analytic and consequently exponentially stable. On the other hand, we prove that the system (1.3) with Dirichlet–Neumann boundary conditions has lack of exponential decay and it decays as $\frac{1}{\sqrt{t}}$ for the case $\gamma_1 > 0,\ \gamma_2 = 0$ or $\gamma_1 = 0,\ \gamma_2 > 0$. Moreover, we prove that this rate is optimal. We apply the main results for the Timoshenko model.

1. Introduction

Due to huge applications of smart materials in modern technology, there has been an abundance of literature on the study of elastic system with viscoelastic damping (see [2]). When a smart material is added in an elastic structure, the Young's modulus, the mass density and the damping coefficients are changed accordingly. Practically, two types of viscoelastic damping are usually used. One is the Botzmann damping and another is the Kelvin–Voigt damping (see [6,7]). These kinds of dampings, on one hand, make the distributed control practically possible, and on the other hand, bring some new mathematical challenges that attract an increasing research interests.

Elastic solids with voids is one of the simple extensions of the theory of classical elasticity. It allows the treatment of porous solids in which the matrix material is elastic and the interstices are void of material. Goodman and Cowin [5] introduced the concept of a continuum theory of granular materials with interstitial voids. Besides the usual elastic effects, these materials have a microstructure with an important property: the mass in each point can be obtained as the product of the mass density of the material matrix by the volume fraction. This idea was developed by Nunziato and Cowin [11] to propose a nonlinear theory of elastic

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materials with voids. That is materials where the skeletal or matrix material is elastic and the interstices are void of material.

With this in mind, in the present work, we are concerned with the analyticity, lack of exponential decay and optimal polynomial decay rate for one-dimensional equations of a homogeneous and isotropic porous elastic solid with Kelvin–Voigt damping. To start, let us consider the following evolution equations in one-dimensional case

$$\begin{cases} \rho u_{tt} = T_x, \\ J\phi_{tt} = H_x + G. \end{cases} \tag{1.1}$$

Here T is the stress, H is the equilibrated stress and G is the equilibrated body force. The variables u and ϕ represent the displacement of a solid elastic material and the volume fraction, respectively. The constitutive equations are

$$\begin{cases} T = \mu u_x + \gamma_1 u_{tx} + b\phi, \\ H = \delta\phi_x + \gamma_2\phi_{tx}, \\ G = -bu_x - \xi\phi. \end{cases} \tag{1.2}$$

Then substituting (1.2) into (1.1), we have

$$\begin{cases} \rho u_{tt} - (\mu u_x + \gamma_1 u_{tx})_x - b\phi_x = 0 \quad \text{in} \quad (0,L)\times(0,\infty), \\ J\phi_{tt} - (\delta\phi_x + \gamma_2\phi_{tx})_x + bu_x + \xi\phi = 0 \quad \text{in} \quad (0,L)\times(0,\infty), \\ (u(x,0),\phi(x,0)) = (u_0(x),\phi_0(x)), \quad \text{in} \quad (0,L), \\ (u_t(x,0),\phi_t(x,0)) = (u_1(x),\phi_1(x)), \quad \text{in} \quad (0,L). \end{cases} \tag{1.3}$$

To this system we add Dirichlet–Dirichlet boundary conditions

$$u(0,t) = u(L,t) = \phi(0,t) = \phi(L,t) = 0, \quad t > 0 \tag{1.4}$$

or Dirichlet–Neumann boundary conditions

$$u(0,t) = u(L,t) = \phi_x(0,t) = \phi_x(L,t) = 0, \quad t > 0 \tag{1.5}$$

where ρ, μ, J, δ, b, ξ, γ_1 and γ_2 are the constitutive coefficients whose physical meaning is well known. The constitutive coefficients, in one-dimensional case, satisfy

$$\xi > 0,\ \delta > 0,\ \mu > 0,\ \rho > 0,\ J > 0,\ \mu\xi \geq b^2, \text{ and } \gamma_1, \gamma_2 \geq 0. \tag{1.6}$$

As coupling is considered, b must be different from 0, but its sign does not matter in the analysis.

It is worth mentioning some papers in connection with the goal of our article. In [15] R. Quintanilla studied the following system

$$\begin{cases} \rho u_{tt} - \mu u_{xx} - b\phi_x = 0 \quad \text{in} \quad (0,L)\times(0,\infty), \\ J\phi_{tt} - \delta\phi_{xx} + bu_x + \tau\phi_t + \xi\phi = 0 \quad \text{in} \quad (0,L)\times(0,\infty), \\ u(0,t) = u(L,t) = \phi_x(0,t) = \phi_x(L,t) = 0, \quad \forall t > 0, \\ (u(x,0),\phi(x,0)) = (u_0(x),\phi_0(x)), \quad \text{in} \quad (0,L), \\ (u_t(x,0),\phi_t(x,0)) = (u_1(x),\phi_1(x)), \quad \text{in} \quad (0,L). \end{cases} \tag{1.7}$$

He proved that the system (1.7) is not exponentially stable and he not show any type of decay rate. In [9] A. Magaña and R. Quintanilla considered the system: