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Hydrodynamic singular regimes in 1 + 1 kinetic models and spectral numerical methods

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ABSTRACT

Classical results from spectral theory of stationary linear kinetic equations are applied to efficiently approximate two physically relevant weakly nonlinear kinetic models: a model of chemotaxis involving a biased velocity-redistribution integral term, and a Vlasov–Fokker–Planck (VFP) system. Both are coupled to an attractive elliptic equation producing corresponding mean-field potentials. Spectral decompositions of stationary kinetic distributions are recalled, based on a variation of Case's elementary solutions (for the first model) and on a Sturm–Liouville eigenvalue problem (for the second one). Well-balanced Godunov schemes with strong stability properties are deduced. Moreover, in the stiff hydrodynamical scaling, an hybridized algorithm is set up, for which asymptotic-preserving properties can be established under mild restrictions on the computational grid. Several numerical validations are displayed, including the consistency of the VFP model with Burgers–Hopf dynamics on the velocity field after blowup of the macroscopic density into Dirac masses.

1. Introduction and theoretical setup

This paper is devoted to the numerical discretization of kinetic systems at hydrodynamic scaling in the strongly attractive setting. We are mainly interested in two systems, one modeling chemotaxis, and the Vlasov–Poisson Fokker–Planck (VPFP) system in plasma physics. Our aim is to propose accurate numerical schemes able to cope with hydrodynamic limit i.e. consistent with the limiting macroscopic model for which solutions may blowup into Dirac deltas in finite time.

1.1. Presentation of the two main kinetic models

The present paper, a followup of [36,32], is devoted to the following two models:

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- a first one describing the motion by chemotaxis, proposed in e.g. [41,17,26]:

$$\partial_t f + v\partial_x f = \frac{1}{\varepsilon}\left(h(v\partial_x S)\rho - f(t,x,v)\right), \quad 0 \le h \le 1, \quad \int_{-1}^{1} h(v\partial_x S)dv = 1, \tag{1}$$

$$-\partial_{xx} S + S = \rho := \int_{-1}^{1} f(t,x,v')dv'. \tag{2}$$

In this model, the distribution function f governs the dynamics of bacteria or cells moving by chemotaxis, i.e. responding to a gradient of a chemical S generating by cells themselves. In this equation $x \in \mathbb{R}$, $v \in [-1,1]$ and $t > 0$. In order to guarantee the condition on h, we may assume

$$h(u) = \frac{1}{2} + h_1(u), \quad \text{with} \quad h_1(-u) = -h_1(u), \quad -\frac{1}{2} \le h_1(\cdot) \le \frac{1}{2}. \tag{3}$$

When taxis dominates the unbiased movements of cells, $\varepsilon \ll 1$, so the dynamics is strongly aggregative. Then, the first moments in v of the solution to the mesoscopic model can be approximated by a simplified macroscopic equation, of aggregation type [21,35]. Indeed, taking the two first moments of (1), we have

$$\partial_t \rho + \partial_x J = 0, \qquad \varepsilon(\partial_t J + \partial_x q) = a(\partial_x S)\rho - J, \tag{4}$$

where it was set

$$J = \int_{-1}^{1} v f(v)dv, \qquad q = \int_{-1}^{1} v^2 f(v)dv, \qquad a(u) = \int_{-1}^{1} v h(vu)dv = \int_{-1}^{1} v h_1(vu)dx, \tag{5}$$

and (3) was used to produce the last equality. We deduce easily (at least formally) from (4), that at the limit $\varepsilon \to 0$, an aggregation equation emerges,

$$\boxed{\partial_t \rho + \partial_x(a(\partial_x S)\rho) = 0, \qquad -\partial_{xx} S + S = \rho.} \tag{6}$$

- A second model of interest in this work is the Vlasov–Poisson–Fokker–Planck (VPFP) system. It governs the dynamics of the distribution function f of charge carriers within an electric field $E = \partial_x \phi$ generated by the charge carriers themselves. The system reads [40]:

$$\varepsilon(\partial_t f + v\partial_x f) + \partial_x \phi \cdot \partial_v f = \frac{1}{\theta}\partial_v(vf + \kappa\partial_v f), \tag{7}$$

$$-\partial_{xx}\phi = \rho := \int_{\mathbb{R}} f(v)dv. \tag{8}$$

The high field regime corresponds to letting $\varepsilon \to 0$. As above, we take the two first moments of (7),

$$\partial_t \rho + \partial_x J = 0, \qquad \varepsilon\theta(\partial_t J + \partial_x q) = \theta\partial_x \phi\rho - J, \tag{9}$$

where we set $J = \int_{\mathbb{R}} v f(v)dv$, and $q = \int_{\mathbb{R}} v^2 f(v)dv$. We deduce easily at least formally from (9), that at the limit $\varepsilon \to 0$ we recover the aggregation equation

$$\boxed{\partial_t \rho + \theta\partial_x(\partial_x \phi\rho) = 0, \qquad -\partial_{xx}\phi = \rho.} \tag{10}$$