J. Math. Anal. Appl. 445 (2017) 710–718

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Separated solutions of logistic equation with nonperiodic harvesting

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ARTICLE INFO

Article history:
Received 16 July 2016
Available online 11 August 2016
Submitted by V. Radulescu

Keywords:
Logistic
Separated solutions
Blow up
Non-periodic
Species
Harvesting

ABSTRACT

In this paper, we study Logistic equations with non-periodic harvesting. We introduce the notion of *separated solutions* and then show the existence of upper and lower solutions that are separated. We further show that the lower solution is unique and all solutions that start above the lower solution tend to each other, while solutions that start below the lower solution blow up in finite time.

1. Introduction

In recent years, numerous studies have been made on the logistic differential equation with harvesting. Consider the equation

$$P'(t) = R\,P(t)\,[1 - \frac{P(t)}{L}] - H(t)$$

where P measures the population of species, t is time, R is the intrinsic growth rate (R constant), L is the carrying capacity (L a constant) and H is the harvesting rate, where H(t) is periodic in t. For further biological interpretation, one might consult [4].

Setting $y(t) = \frac{P(t)}{L}$ and $A(t) = \frac{H(t)}{L}$, we obtain

$$y' = R(t)y[1 - y] - A(t).$$

Finally, letting $z = Ry$ and $h(t) = A(t)R$, the above equation is changed to

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http://dx.doi.org/10.1016/j.jmaa.2016.08.013

$$z' = z[R - z] - h(t).$$

In this paper we study equations of the form

$$y' = a(t)y(t) - y^2(t) - b(t) \tag{1.1}$$

where $a(t)$ and $b(t)$ are assumed to be continuous and positive. In the case where $a(t)$ and $b(t)$ are periodic, this equation or more general equations containing it have been extensively studied, see [4–8], and their bibliographies. We point out that the methods used for the periodic case are not applicable in our case. The non-periodic case, which seems to be more complicated, has not been studied as extensively. One of the first attempts in this direction was made in [1]; see also [2,3,10].

A fruitful and key concept that we introduce here is that of two *separated solutions*. We say that two solutions y_1 and y_2, both positive and bounded on $[0, \infty)$ $(y_1 t) > y_2(t))$, are *separated* if

$$\int_0^\infty [y_1(t) - y_2(t)]\, dt = \infty.$$

We call the smaller of the two solutions the *lower solution* and the larger one the *upper solution*. It turns out that the lower solution is, in a sense, unique while the upper one is not. Furthermore, we show that if there exist two separated solutions, then any solution that starts above the lower solution tends to the upper solution, any solution that starts below the lower solution blows up to $-\infty$ in finite time, and solutions that start above the upper solution tend to each other.

We also consider a parameter-dependent differential equation by letting $b(t) = k\gamma(t)$, where k is a positive constant and $\gamma(t)$ is positive, continuous and bounded above and below by positive constants. We show that there exists a constant $\bar{k} > 0$, such that for $0 < k < \bar{k}$, the differential equation (1.1), corresponding to k, has two separated solutions, and for $k > \bar{k}$ there does not exist a positive bounded solution. For $k = \bar{k}$, there exists a positive bounded solution, and *we conjecture that there do not exist two separated solutions.*

In the periodic case, it has been shown, see, [1,6], that for $0 < k < \bar{k}$ there exist exactly two positive periodic solutions, for $k > \bar{k}$ there is no positive bounded solution, and for $k = \bar{k}$ there exists exactly one positive periodic solution. It can be easily shown that our results imply these assertions also in the periodic case (see Remark 4.1).

We might point out that we have made a concerted effort to make the proofs simple enough to be understood not only by mathematicians who may not be familiar with this area of research but also by other scientists and engineers with sufficient knowledge of Analysis and Differential Equations (see [9]). Thus the proofs are fairly elementary but require mathematical maturity.

2. Separated solutions of harvested logistic equation

With the motivation given in the Introduction, we now study the differential equation

$$z' = a(t)z - z^2 - b(t) \tag{2.1}$$

where we assume $a(t)$ and $b(t)$ to be continuous and bounded above and below by positive constants on $[0, \infty)$. For convenience, we first state a known lemma the proof of which is an easy exercise in Calculus.